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ARTIFICIAL INTELLIGENCE

Intelligent Control Systems. I. Quantum Computing and Self-Organization Algorithm

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Abstract—The role and methodology of application of quantum computing in problems of design of robust intelligent control systems under the conditions of contingency control situations are discussed. The quantum self-organization control algorithm, which contains the self-organization control algorithm for robust knowledge bases as the particular case, is developed. The satisfaction of the introduced thermodynamic criterion (in the form of the minimum of the generalized entropy in the established relation between such qualitative control characteristics as stability, controllability, and robustness) is the goal of application of the quantum self-organization control algorithm for knowledge bases used in the control loop. The essentially nonlinear and globally unstable control object is used as the example for application of the quantum self-organization control algorithm to the solution of the vector optimization problem based on the decomposition of knowledge bases.

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INTRODUCTION. PROBLEMS OF DEVELOPMENT OF INTELLIGENT CONTROL SYSTEMS AND GOAL OF OPERATION

One of the main designations and advantages of efficient application of intelligent control systems is the possibility of guaranteed achievement of the control target with maximal control quality on the higher level and minimal consumption of the useful resource of the "control object + controller" system on the lower (actuating) level of the hierarchical automatic control system [1]. On the conceptual level this possibility reflects the designation of the purposeful activity of the intelligent control system in the general case of contingency control situations. In this case the efficiency of application of the intelligent control system depends on the level of intelligence [2] of the developed system (form, type, and deep knowledge representation [3]). The choice of the applied tools of the technology of intelligent computing[1] for designing the corresponding knowledge base for the given control target plays the important role upon formation of the level of intelligence of the automatic control system.

[1] The determination and the term "intelligent computing" were introduced in the mid-1990s [4]. The technology of intelligent computing was developed in new forms of evolutionary programming, optimization algorithms of the type of immune algorithms, based on behavioral reactions of a swarm (people in a tunnel, ant colonies, bird flocks, fish shoals, animal hordes, etc.), quantum genetic algorithms of global optimization and quantum learning neural networks, and so on.

A. *Role of intelligent control systems.* The development of intelligent control systems for conventional automatic control systems with increased robustness capable of preserving the required levels of precision and reliability in the contingency control situations became the intellectual property [5]; it is of great theoretical, practical, and commercial importance. The software and hardware support of intelligent control systems became a trading product upon commercialization of the intellectual product in knowledge-based engineering management [6]. In particular, the important direction of applied use of intelligent control systems is the increase in the *robustness* of conventional (P/PI/PD/PID) controllers used on the lower levels of the control loop.

Remark 1. According to [5], the conventional PID controller in the control theory is used in more than 85% of automatic control systems. Therefore, one of the important particular (theoretical and practical) problems in creation of intelligent control systems is the development of methods and algorithms for increasing the reliability and quality of control of the actuating (lower) level of the automatic control system based on the conventional PID controller. In this case, following [1, 7], the principle of "nondestruction of the actuating level" is satisfied, which results in the additional efficient application of the existing well-adjusted technological processes and higher economic effect. The application of fuzzy controllers together with the PID controller resulted in the creation of hybrid fuzzy intelligent control systems with different levels of intelligence depending on the completeness and correctness of the designed data base [2].

Here, the term data base is used in the meaning standard for the theory of fuzzy controllers [2, 4] as the finite set of production rules (look-up table) of a certain model of fuzzy inference with particular types and parameters of membership function forming the control laws for the control object. The parameters and type of the membership function are stored in the data base of the fuzzy controller. The structure of the fuzzy self-organized PID controller with a certain level of intelligence [2] was introduced in [8]; it is used in this paper.

R e m a r k 2. For complete representation the following example is given in Section 4: the design of the knowledge base in the form of production rules "If **A** then **B**", where **A** and **B** are the linguistic variables with particular membership functions formed using the knowledge base optimizer on soft computing for the dynamic control object cart-pole. This example is also used to illustrate the quantum fuzzy inference for formation of robust control signals for gains of the fuzzy PID controller due to self-organization of knowledge bases designed earlier (using the knowledge base optimizer).

B. *Problems of development of intelligent control systems.* One of the difficulties of development of intelligent control systems for contingency control situations is the solution of the problem of corresponding robust knowledge base design [1] using objective knowledge on the dynamic behavior of the control object and fuzzy PID controllers. The solution of this problem considerably depends on the possibility of development of algorithmically solvable, physically/mathematically correct model [9] and the tools for practical implementation of the process of extraction, development, and formation of objective knowledge without the participation of an expert [4, 7]. The introduction of physical and information constraints into the formalized description of the model of the control object [1, 9] considerably influences the quality of the formed knowledge base in the intelligent control system, and the elimination of these constraints from the description of the models of control objects results in the incorrectness and loss of robustness of the designed control laws, respectively [10].

Therefore, one of the complex key problems of the development of the basis of information technology of intelligent control systems design for such a wide class of control objects is the creation of the process of designing robust knowledge bases for contingency control situations for the actuating level of the hierarchical control structure taking into account real physical and information constraints in the production rules of the knowledge base [8].

C. *Design technology and structure of intelligent control systems.* Figure 1a shows the typical structure of the intelligent control system and describes the problem of objective knowledge base design considered in this paper. Figure 1b shows the structural diagram of the information technology and design stages of the objective knowledge base for robust intelligent control systems based on new types of intelligent computing.

Note that the output signal of the fuzzy controller in Fig. 1a determines the vector $K = \{k_P, k_D, k_I,\}$ of gains of the PID controller and $\{k_P, k_D, k_I,\}$ means the proportional, differential, and integral gains, respectively.

The crossed block "Expert system" means that the developed technology of robust knowledge base design does not use expert subjective estimates of the production rules in the knowledge base which occur in the case of interpretation of the results of measurements and observation of the output signal X of the control object (Fig. 1a).

R e m a r k 3. Objectively the measured signal X is the initial information for the expert, and the evaluation of this information by the expert upon construction of the knowledge base in the general case (for example, in the case of large dimensionality) has the subjective character. The technology of intelligent measurements can be directly applied to results of measurements and extract the knowledge from them without subjective interpretation due to the fitness functions in the genetic algorithm. We underline that in this paper the standard notation of the structural diagrams of intelligent control systems is used [1, 2, 7].

Therefore, the task of the design technology (denoted in Fig. 1a and structurally presented in Fig. 1b) is the formation of objective robust knowledge base in the intelligent control system for contingency control situations.

R e m a r k 4. In Fig. 1 (and Fig. 2 below) and in the text below the following notation is introduced: r.s. is the reference signal (disturbances of the control object) reproduced by the forming filters [7]; U^* is the control force with the imposed noise; X is the output signal of the control object; ε is the control error; $K(t)$ are the gains of the PID controller; GA is the genetic algorithm; FC is the fuzzy controller, QFI is the quantum fuzzy inference, KBO is the knowledge base optimizer; GIF is the global intelligent feedback; QA is the quantum algorithm; and CO is the control object. Dashed frame of the block "Control law" does not include the PID controller, although the control signal u is formed by this device (Fig. 1a).

The laws of formation of the control force $u(t)$ follow from the structure of the PID controller and are well known. This fact is the advantage of PID controllers. The solutions for time formation of optimal control laws by the gains of PID controllers are less known, in spite of a rather large number of publications (more than 85 patents on formation of control laws for gains of PID controller are known).

Therefore, methodologically in this paper, due to the extensive application, the PID controller is chosen

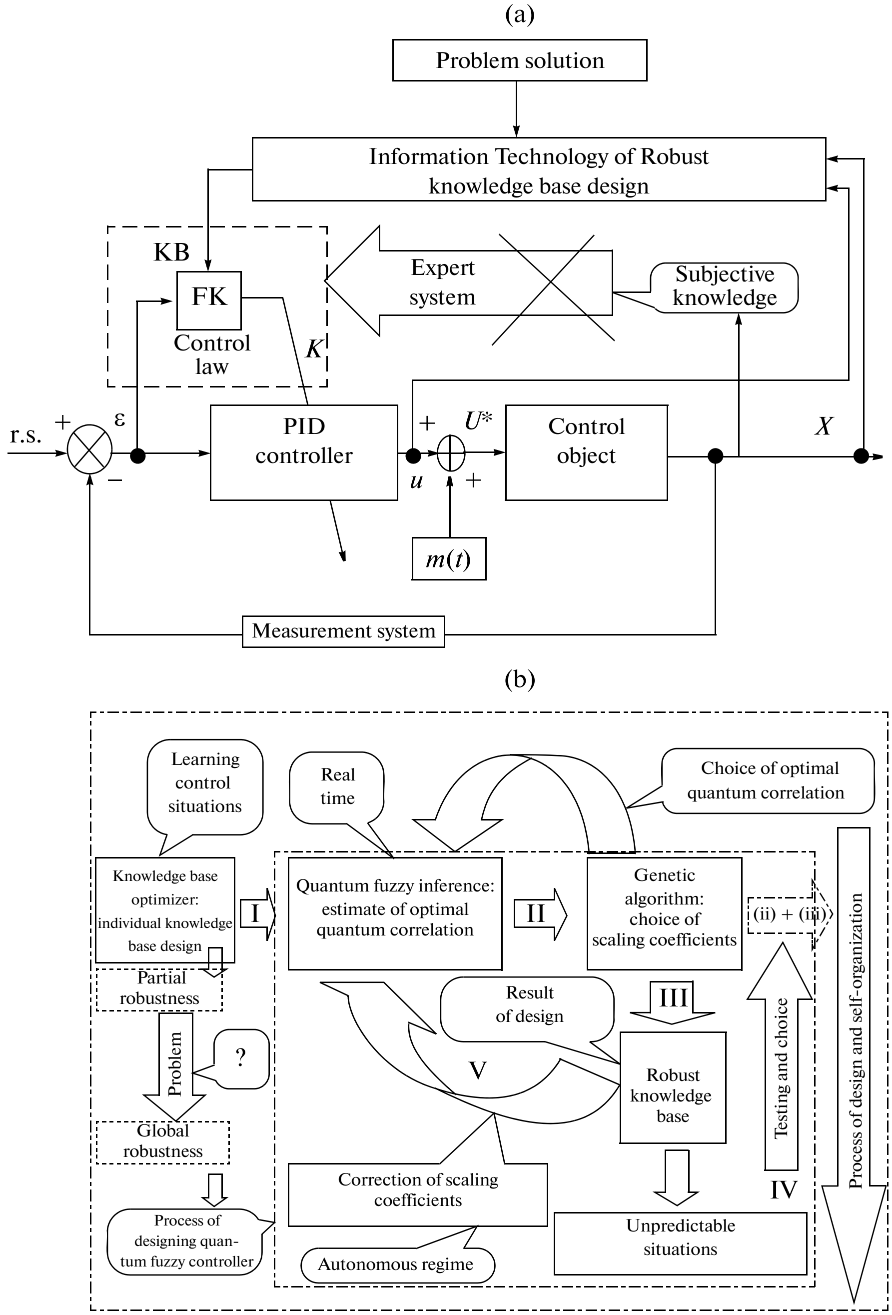


Fig. 1. Design of robust knowledge bases in contingency control situations: (a) knowledge base design problem; (b) structure of technology and design stages.

as the initial one and this block is selected with the term "Control law" applied to it. Note that many controllers with sliding regimes, automatic control systems with variable structure, fractional or noninteger order controllers, etc. were studied by us and it was demonstrated [11] that they are also efficiently designed using the developed technology presented in Fig. 1. The simplicity of physical implementation of the designed control laws for the actuating level together with the increased "level of intelligence" yields the necessary technical effect [10, 11]: nondestruction of the actuating level with the guaranteed control efficiency and reliability due to the increased level of its intelligence.

Let us briefly describe the main stages of information technology of robust knowledge base design presented in Fig. 1b.

At stage I of design using the technology of soft computing (the block of knowledge base optimizer) the finite set of knowledge bases for particular control situations marked in Fig. 1b by the note "Learning control situations" is formed. At this stage, as shown in [7, 8], the problem of formation of the partial robustness of the intelligent control system is solved; in this case, the intelligent control system executes the control problems for the given class of control situations. The question of formation of the property of global robustness of the intelligent control system (denoted in Fig. 1b by the note of the question mark) is the problem solved by this information design technology.

Successive transition (from partial to global robustness) results in the process of designing the quantum fuzzy controller using quantum strategies in the logical fuzzy inference (in the form of the quantum fuzzy inference denoted by corresponding note in Fig. 1b). This transition is implemented at stages II, III, and IV, respectively. After obtaining the results of operation of the quantum fuzzy inference the block of genetic algorithm forms the scaling factors for the generalized gains of the PID controller. The results (ii) $\in$ are related to stage II, and (iii) $\in$ to stage III. The other substages of the design process are shown by corresponding notes in Fig. 1b; their interpretation is given below. The process of designing self-organization of robust knowledge bases in contingency control situations is implemented via the total action of the above design stages.

The structures of robust intelligent control systems based on the developed design technology (see Fig. 1b) are shown in Fig. 2.

Note some specific features of the notation in Fig. 2.

The two-sided arrow "Entropy production" in Fig. 2b denotes the process of calculation of the entropy production by signals X and u, which is equivalent to the presence of the calculation block "Entropy production" in both connections. This process was described in detail in [2, 7, 8]. The entropy production is considered in the genetic algorithm as one of the components of the fitness function (the criterion of the minimum of generalized entropy, see Fig. 6a) and is the thermodynamic quantity. Therefore, the correct operation with physically homogeneous variables (according to the laws of thermodynamics of open systems) is executed in the integrator in Fig. 2. The example of application of this operation is considered in Section 4.

In the model of quantum fuzzy inference in Fig. 2b the quantum knowledge hidden in classical states obtained at the output of the block "Formation of objective knowledge based on soft computing" is extracted. In contingency control situations the quantum fuzzy inference based on the reaction of production rules of the robust knowledge base designed by the fuzzy controller (see Figs. 13 and 18 below) forms and implements in real time the control laws with account of nonlinear physical and information constraints on the conditions of operation of the control object. As a result, the quantum fuzzy inference is used to design the robustness of the intelligent control system in the control laws for the gains of the PID controller including the mentioned specific features.

The technologies of soft and quantum computing [4, 12–15] are used as the tools for simulation of production rules of knowledge bases.

D. *Goal of the study.* In this paper consisting of two interconnected parts the solutions of two problems are considered. Part I presents the quantum algorithm of knowledge self-organization control and the role of analogues of physical (quantum and thermodynamic) effects in the implementation of the process of guaranteed achievement of the control quality. In this case the main attention is paid to the description of the qualitative specific features of the biologically reproducible evolution of self-organization whose basic components are described by quantum operators and comprise the content of the developed model of the quantum algorithm of self-organization control. The problem of robust control in contingency situations based on quantum strategies of making decisions in the form of quantum fuzzy inference as the particular case of the developed generalized quantum algorithm of self-organization control is considered. Part II presents the results of application of the technology of robust knowledge base design in the structure of the intelligent control systems (see Figs. 1b and 2b).

E. *Applied aspects.* The developed software tools of new intelligent computing (unconventional computational intelligence [15]) includes the principle of self-organization of the knowledge base based on the quantum approach and implements the thermodynamic criterion of optimal ratio of the control features, such as the stability, controllability, and robustness.

The quantum algorithm introduced in this paper is the new quantum search algorithm. The solution of the problem of designing the robust control of the control object (locally or globally unstable and essentially nonlinear) with the vector criterion of control quality is considered in this paper as the example (benchmark) of efficient use of the developed information design technology.

In Section 4 the possibility of efficient application of the technology of quantum computing is illustrated, in particular, for solution of such algorithmically unsolvable by classical methods problems as designing the global robustness of intelligent control systems in the conditions of contingency control situations [10, 16]. Classical methods mean methods of recurrent stochastic optimization, etc., included, for example, in the industrial software for support of the system of engineering calculations of the type of MathLab and SimuLink.

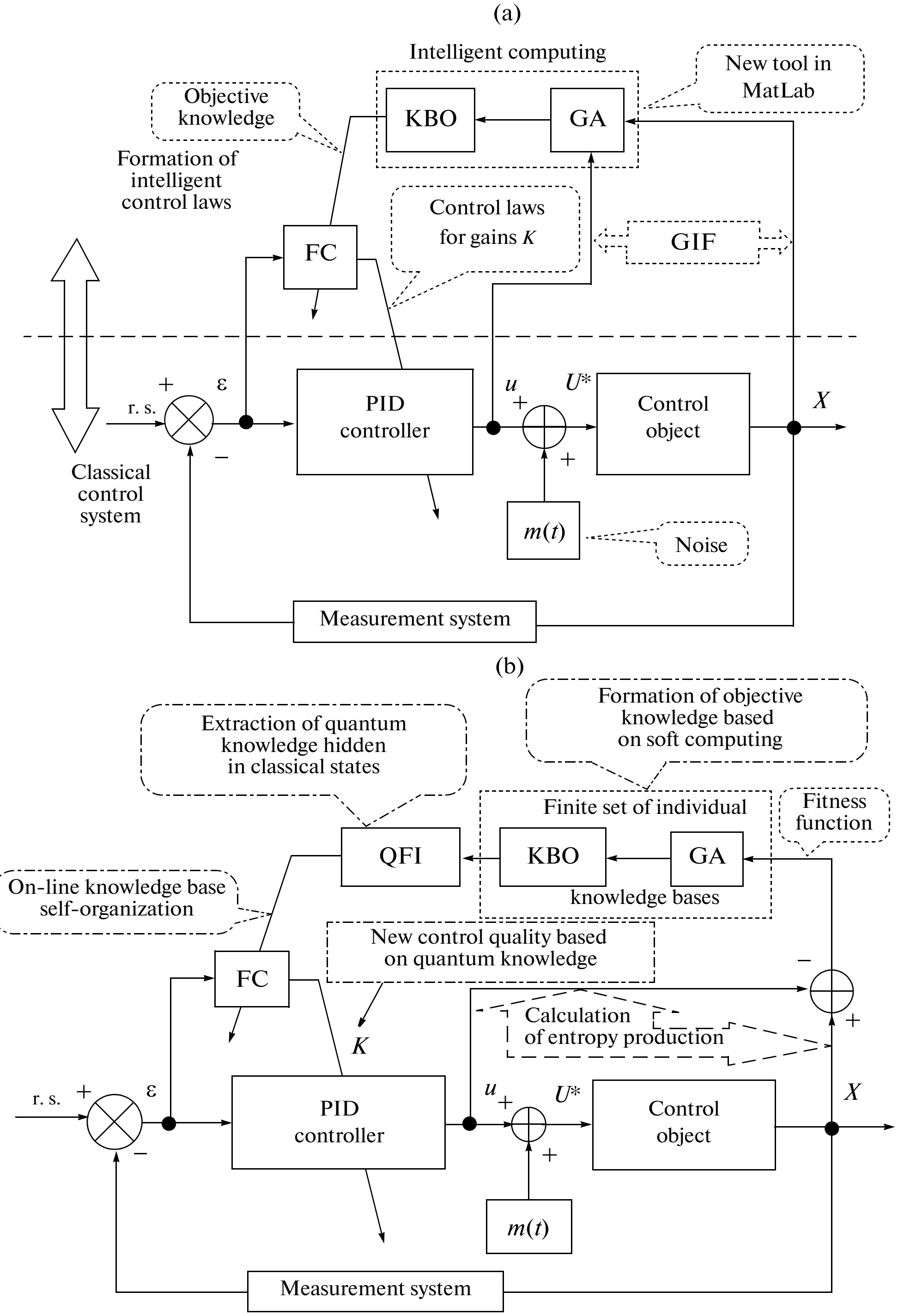


Fig. 2. Structures of intelligent control systems with knowledge base optimizers with intelligent computing: (a) structure of intelligent control system with knowledge base optimizer with soft computing; (b) structure of intelligent control system with knowledge base optimizer with quantum computing; GIF is global intelligent feedback.

1. ROLE AND PRINCIPLES OF TECHNOLOGIES OF INTELLIGENT COMPUTING IN DESIGN OF INTELLIGENT CONTROL SYSTEMS

In conventional calculus the qualitative property of the studied object is judged by the quantitative result of the numerical algorithm. The logical evaluation of the property in the general case can be made only at the end of quantitative calculations, and for algorithmically unsolvable numerical problems (algorithm complexity according to Kolmogorov [9]) often the sought estimate is unachievable.

The basis of intelligent computing is not only numerical scales (as in conventional calculations) but also qualitative characteristics of the studied object. The goal and the possibility of intelligent computing is the direct determination of the qualitative characteristics of the object by algorithmic operation of these characteristics similar to numerical scales. Quantitative estimates of the object can be obtained from the qualitative estimates by inverse representation of scales. Soft and quantum computing are the examples of such calculus. In this case, many classical problems can be solved with exponential acceleration and algorithmically unsolvable classical problems can be solved [11–15] using quantum computing.

Thus, for example, for calculation of the qualitative characteristic of the function (constant or balancing for four values of the argument) in conventional approach four steps of numerical operations are required, while Deutsch's quantum algorithm determines this quality of the studied function in one step. Schor's algorithm solves the factorization problem for the given number with exponential rate with respect to the best known algorithm and if the length of the number increases this algorithm solves the algorithmically unsolvable problem with the polynomial complexity.

Grover's algorithm seeks the solution in unstructured data with quadratic speed-up. The complete description of these quantum algorithms is given in [11–14].

Fuzzy sets, introduced in 1965 by L. Zadeh, extend the determination of the number (used during many centuries) and introduce the set of new scales of qualitative characteristics which cannot be determined using classical calculus. This calculus opens new possibilities for the theory and design of intelligent control systems.

Nonstandard logics used as the basis of intelligent computing and inferences obtained using them in decision making and control problems often result in the imaginary "paradoxes" and contradiction with intuitive ideas of the research engineer concerning the expected result. The term "nonstandard" logic already includes this situation, and its introduction is justified by the desire of solving problems which cannot be solved by existing calculation technologies. These technologies of intelligent computing include soft and quantum computing used in this paper for robust knowledge base design in the conditions of contingency control situations. The application of new technologies in engineering practice of the control theory and systems often encounters the problems of overcoming the inertia of the "pragmatic" intuition and engineering philosophy. This took place in the mid 1970se, as the ideas of soft computing based on the theory of fuzzy sets in the form of fuzzy control systems were introduced into the engineering practice [1].

Let us consider briefly the methodological specific features of application of the technologies of soft and quantum computing.

1.1. Technology of Soft Computing

The generalization of the idea of the number due to the introduction of the subjective qualitative scale (and the reflection of its quantitative characteristic in this scale in the form of the linguistic approximation) resulted in the 30-year long discussion with representatives of the scientific school of the probability theory. The difficulties include, for example, the correct determination of the notions of the membership function, logical correlations "fuzzy/random quantity", and so on.

The basis of the technology of soft computing is *fuzzy logic* in which the *law of excluded middle* is not used. This results in the nonstandard inference on the possibility of simultaneous consideration of, for example, number 10 on the scale [0, 100] as the linguistic variable "large" or "small" with different values of the membership function on the given qualitative linguistic scale. Only practical application of fuzzy automatic control systems for classical control objects explained and removed the divergences and difficulties of determination of individual and mass events, operations of averaging and extraction of information, and so on.

As a result, fuzzy models of logical inference in the phase space of linguistic variables provided the development of fuzzy intelligent control systems which efficiently solve control problems in the conditions of essential uncertainty of initial information, weak formalization of description of the control object, and uncertain control targets [1].

One of the main problems of practical and efficient application of the technology of soft computing in control problems was the solution of the following tasks: (a) objective determination of the form of the membership function and its parameters in production rules of the knowledge base; (b) determination of the optimal structure of fuzzy neural networks in learning problems (approximation of the learning signal with the required error and the minimal number of production rules in the knowledge base); (c) application of genetic algorithm in multicriteria control problems and the presence of constraints on the parameters of the control object, etc.

These problems were solved and tested based on the developed knowledge base optimizer using the technology of soft computing. The developed intelligent tools provided the design of robust knowledge bases upon the solution of one of the algorithmically unsolvable (or hardly solvable) problems of the theory of artificial intelligence, the extraction, processing, and formation of objective knowledge without the use of expert estimates. This problem was considered in detail in [7]; in this paper the structure of the knowledge base optimizer, the solved optimization problems, and this problem formulation were described in detail. According to Fig. 1b, the output data of the knowledge base optimizer at stage I are the input data

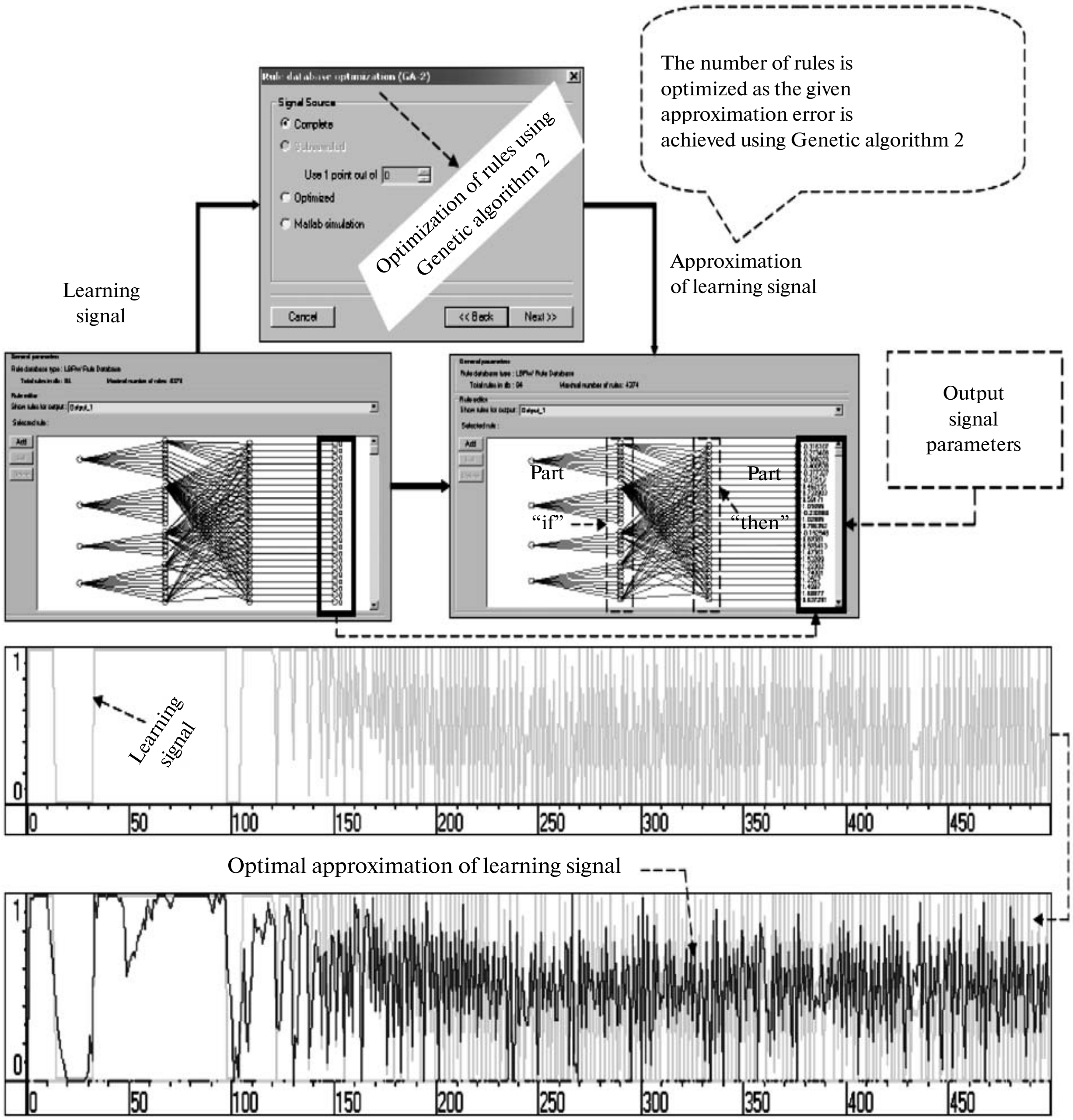


Fig. 3. Example of optimization of the number of production rules upon approximation of the learning signal with given approximation error.

for stage II using the quantum fuzzy inference for the robust knowledge base design.

Note some specific features of the knowledge base optimizer at stage I.

In this optimizer three genetic algorithms are used. Physically the first genetic algorithm optimizes linguistic variables in the left-hand side of the production rule of the type "If **A**..." (see Fig. 3), eliminates the excessive information in the learning signal, and realizes the choice of the optimal power of term sets of linguistic variables of components in the learning signal. The second genetic algorithm optimizes the bases of production rules and chooses the optimal parameters of the right-hand sides of production rules of the type "...then **B**". The third genetic algorithm adjusts the knowledge base. As a consequence, the application of these genetic algorithms provides the optimal structure design of the fuzzy neural network (form of membership functions and their parameters, number of internal layers, etc.) approximating the learning signal with the required error [7].

Figure 4 shows the result of operation of the second genetic algorithm in the knowledge base optimizer upon designing the control laws for the gains of the fuzzy PID controller used in the example of simulation of the system "moving cart—upturned pendulum" (see Section 4.1).

The combination of the methodologies of stochastic and fuzzy simulation of intelligent control systems as the tools of the knowledge base optimizer (developed by us) provided higher robustness of the designed knowledge bases and the solution of complex problems of formation of objective knowledge.

However, in the case of considerable variation or contingency control situations the designed control laws do not always preserve the robustness property [7, 8]. This effect is formed by the functional structure of genetic algorithms in which (by definition) the solution search space is fixed and is determined by the expert and the choice of the fitness function considered as the control optimality criterion. The expert's opinion in the general case is manifested in his experience of correct determination of the search space of the genetic algorithm and the knowledge of the form of the fitness function.

Thus, the optimal solution found using the technology of soft computing (based on the genetic algorithms) corresponds to the given control situation, contains (implicitly) the subjective character of the initial information, and in the case of incorrect determination of the search space and fitness function it can be inadequate to the given control situation.

Note that the solution of the problem of automatic control systems design for extraordinary situations using additional control loops can be related to the mid-1980s; it resulted in the occurrence of the excessive information in control loops [17]. As a consequence, this results in the lower reliability of automatic control systems. Note also that in practice modern PID controllers are more and more often realized in the program way on a new modern elemental base (spintronics); therefore, they possess increased reliability. This problem was studied in detail by many researchers.

The design of control in complicated situations using the simple PID controller with increased level of intelligence using the technology of intelligent computing is one of the possible solutions of such problems. Therefore, for searching optimal solutions for such control situations it is necessary to apply new technologies of intelligent computing, such as the technologies of quantum and soft computing [10]. It should be noted that the technology of soft computing is efficiently used in control problems for quantum control objects [11]. However, until now the application of quantum computing for efficient solution of classically algorithmically unsolvable problems of the control theory and systems encountered the following statement made in [13, 14] (introduced in the early 1980s): that it was necessary to apply quantum computing for the solution of quantum problems only.

1.2. Technology of Quantum Computing

In *quantum logic* of the technologies of intelligent quantum computing and quantum information theory the classical *distributive law* is not satisfied, which reflects new (unusual) phenomena of the type of noncommutative character of variables, uncertainty, and the impossibility of simultaneous precise measurement of observables in quantum mechanics. As a result, unusual phenomena for classical physics, such as entangled states, teleportation, and superdense coding result in "paradoxes" and difficulties of physical interpretation from the point of view of logic of classical physics. Thus, for example two subsystems each of which is in the entangled chaotic state (with nonzero information entropy) in the case of merging into one system form the pure (with the zero von Neumann entropy) entangled state which possesses higher order level (quantum self-organization effect). Quantum superposition consisting of two classical *mutually exclusive* logical states provides the formation of one whole state which contains simultaneously, for example, the logically contradictory "yes" and "no" (Schrödinger's cat). Two classical single-orbit states can be used to additionally (depending on the type of quantum communication) extract more than one bit of additional information using quantum correlation (which is above the classical one). Quantum decision making strategies provide the formation of the player—winner (Parrondo's effect) using the quantum approach to the problem solution [10–14] from two classical players who do not possess winning strategies in the particular game situation.

Thus, the quantum—language description of formulations of many classical (weakly structured) engineering problems (which are difficult to be solved in terms of classical logic) provides their efficient solution. However, this approach has a number of specific features in the case of practical application in control problems. In the theory of quantum information and quantum computing, the notion of the number corresponds to the notions of *observable* and *superposition* of the state of observables of the quantum system, and the irreversible measurement yields one of possible states [12–14].

Figure 5 shows the physical difference in the definition of the calculational basis of soft and quantum computing.

The definition of the notion of the calculational basis in the theory of quantum computing is given briefly in Appendix.

It can be seen from Fig. 5a that the fuzzy state of the number has *two values* of the membership function on the scale of linguistic description (the principle of excluded middle is not satisfied) and the quantum state consists of *two classical states* whose qualitative

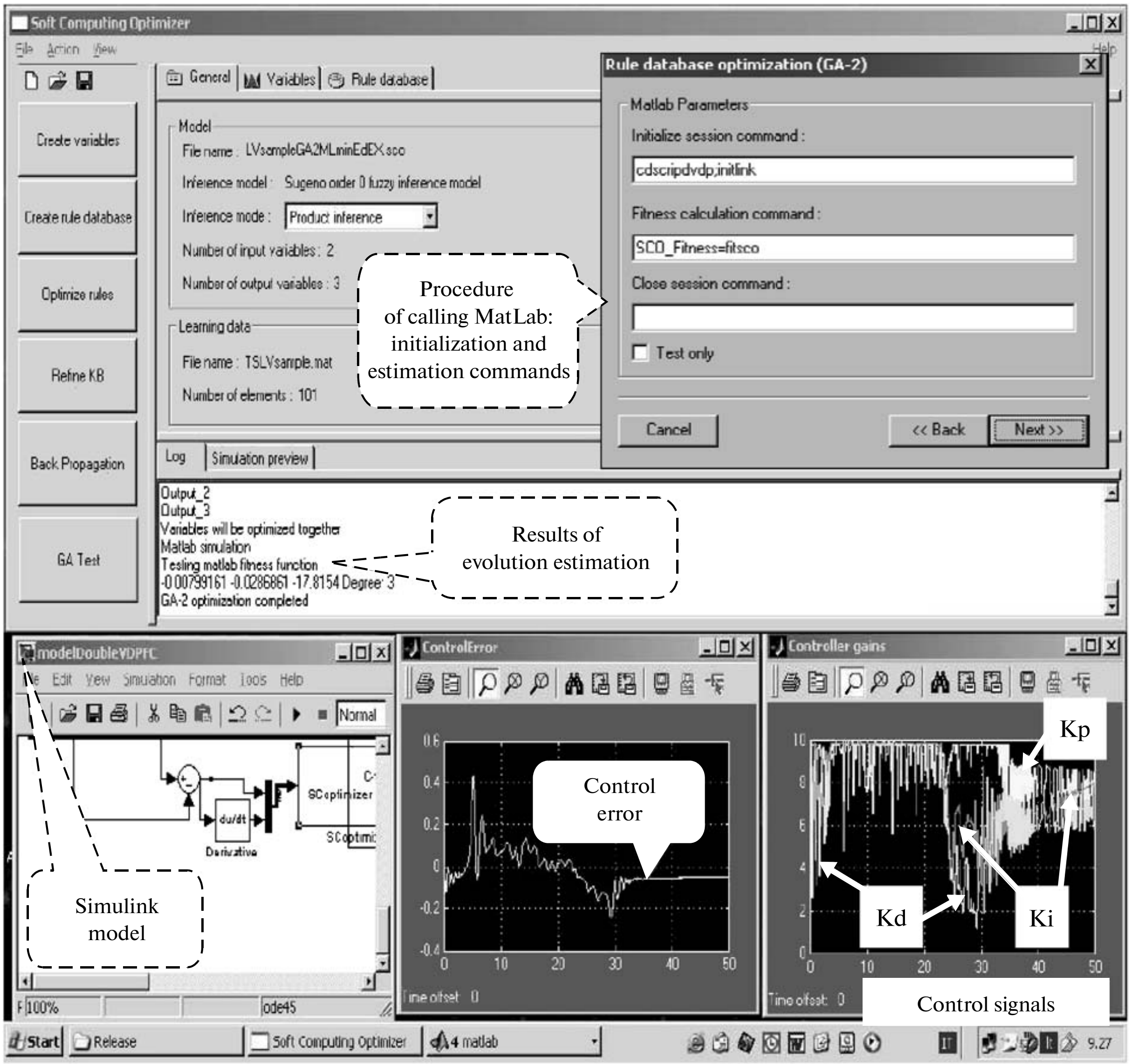


Fig. 4. Example of designing control laws for gains of the fuzzy PID controller with the given approximation error of the learning signal.

characteristics are coded in quantum variables connected by the principle of quantum logical complementarity. In this case the quantum state can have both positive and negative values of the probability amplitude (Fig. 5b), which considerably differs the quantum probability theory from the axiomatics of the classical Kolmogorov's probability theory. Note that the possibility of existence of negative values of the classical probability in quantum entangled states was considered in many papers devoted to quantum mechanics. Therefore, conventionally in papers concerning the quantum probability theory the comparison and differences of Kolmogorov's axiomatics from corresponding definitions of the quantum theory [12] were considered.

The possibility of combined application of calculational bases illustrated in Fig. 5 results in the new type of intelligent computing, quantum soft computing [15, 16, 18–21].

The positive results of application of classical technologies of intelligent computing (of the type of soft computing), together with quantum computing resulted in the new alternative approach, the application of the technology of quantum intelligent computing in the problems of optimization of the control processes for classical control objects (the physical analogue of application of the inverse method of investigation "quantum control system—classical control object") [16, 19, 20].

This approach considerably extends the capabilities of intelligent computing [11].

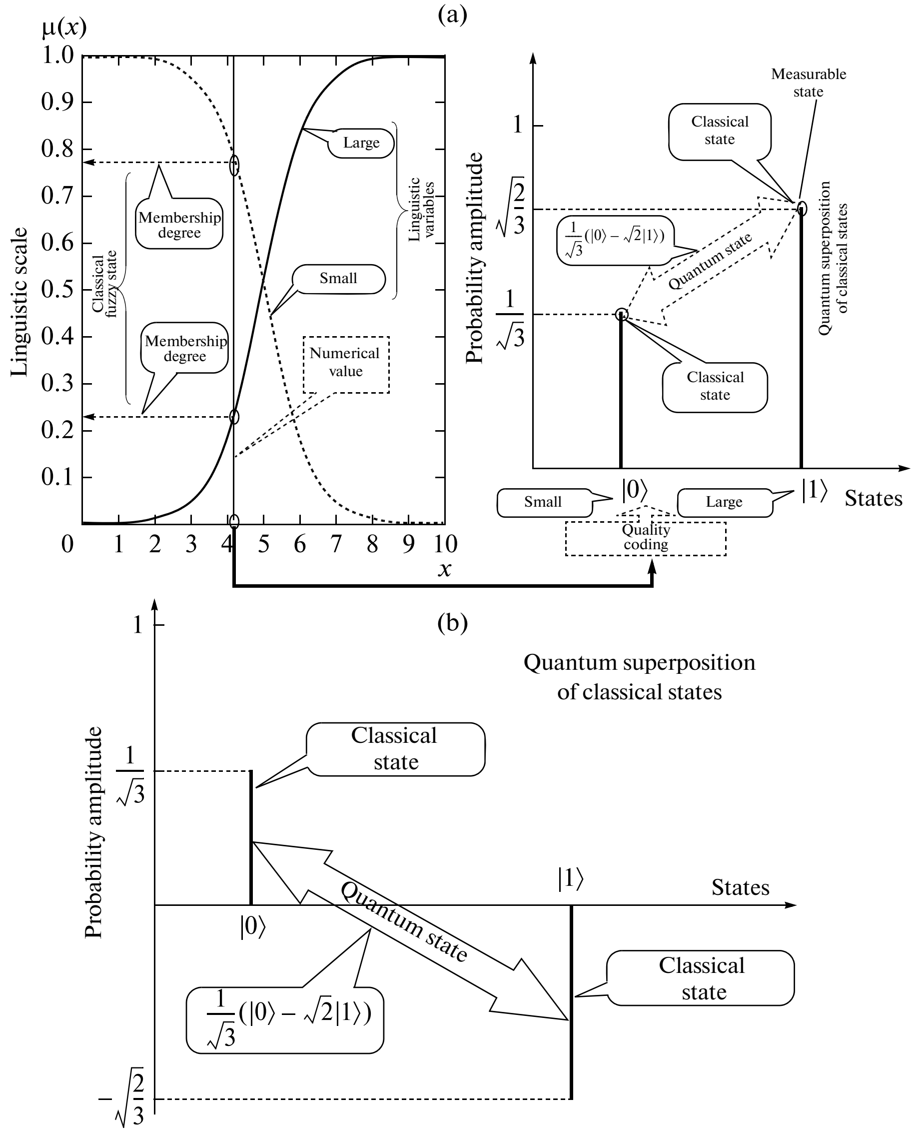


Fig. 5. Physical difference between fuzzy and quantum states: (a) comparison of fuzzy state and quantum superposition; (b) quantum superposition states with negative probability amplitude.

2. FORMULATION OF THE PROBLEM AND SPECIFIC FEATURES OF THE METHOD OF SOLUTION

In this paper, the following problem is considered: develop of the methodology and basis of information technology of self-organized intelligent control systems design. The algorithmic basis of this technology is the quantum algorithm of self-organization control for the knowledge base.

R e m a r k 5. The process of self-organization of robust knowledge bases means the process of designing the fuzzy controller forming at the output robust signals of adaptive control of parameters (gains) of actuating devices which determine the control force of the control object. In this case the reaction of production rules of robust knowledge bases designed earlier (for given control situations) for fuzzy PID controllers in the case of control error and conditions of uncertainty of the new initial information and contingency control situations is used. The designed self-organization process is implemented in real time due to the extraction of the additional quantum information and the reduction of excessive information hidden in classical states of the control process.

Quantum fuzzy inference developed based on four facts of the quantum information theory in [10, 16, 18–20] is the particular case of the self-organization algorithm [15, 18]. The combination on the structural level of the finite number of knowledge bases designed earlier and the model of the quantum fuzzy inference in one block of quantum fuzzy controller provides the possibility of formation in real time of the robust control from the reactions of production rules of these knowledge bases on contingency control situation. In this case, it is sufficient to use just minimal information on the change of the control situation contained in the new control error (see description of Figs. 13 and 18 below).

Thus, the control algorithm for self-organization of knowledge bases is used to extract hidden additional quantum information from classical information contained in the reaction of classical states of control signals for parameters (gains) of actuators designed earlier for the finite number of learning situations [18]. In this case excessive information in classical states of control signals is reduced [17, 20].

The result of operation of the control algorithm is the self-organization of the knowledge base which is used to choose the appropriate gains of the PID controller. Thus, the robust control signal is formed in real time for the parameters of the corresponding actuators which determine the control force for the control objects in the conditions of contingency control situations.

Remark 6. In order to avoid misunderstanding in interpreting the self-organization process it should also be noted that physically by definition [11, 22–24] "self-assembling" and "self-organization" are different notions. The "self-assembling" process differs from "self-organization" by the absence in the general case in the "self-assembling" of preliminary choice of the type of correlation between non-connected elements ("building blocks") and by the fact that the process is performed due to local interaction and external (in the general case random) impacts [22]. The control of the choice (of the type and form) of quantum correlation between the "building blocks" in the "self-assembling" process determines the form of the synergetic cooperation in the structure formed due to self-organization [24–26]. Therefore, the introduction of the control of the choice of correlation provides the account in the formed structures of the synergy of purposeful cooperative effects from the limited number of "building blocks" and makes the basis of the self-organization process [23, 25, 26].

The additional physical aspects of self-organization, details, and technical features of self-organization control can be found in [15, Section "Design Technology and Self-Organization of Robust Knowledge Bases".

The solution of this task is directly connected with the following (difficult and fundamentally important for the control theory and systems) problem: *the determination of the role and influence of analogues of quantum effects on increasing the level of robustness of designed intelligent control systems*.

According to Fig. 1b, the central idea of the effective application of the technology of the processes of extraction, processing, and compressing valuable information for formation of active knowledge in the form of robust knowledge bases is the *structured* random search (based on the technology of soft computing, Stage I—the formation of the finite set of individual knowledge bases in particular learning control situations). The application in the developed algorithm of *quantum control strategies of knowledge self-organization* makes the essence of Stage II at which based on the quantum fuzzy inference the self-organization of (real-time) active knowledge from the reactions of designed individual knowledge bases on the new contingency control situation takes place.

Self-organization levels in the hierarchy of cognitive evolution control processes were analyzed from the point of view of the intelligent system of engineering systems as the new applied branch of the theory of artificial intelligence [16]. It was demonstrated [19, 20] that for guaranteed achieving the control target the control laws for the change of the gains (coefficient gain schedule) of the fuzzy PID controller should be designed based on additional extracted knowledge providing the recognition of physical specific features and information constraints for the control object. Extracted knowledge is used for the self-organization of this control object for achieving the control target in the particular control situation.

The formulated problem is solved based on the technology of soft and quantum computing. In the developed information technology the new principle [16] of designing robust structures of intelligent control systems is efficiently realized: *the design of a fuzzy controller with increased level of intelligence (wise controller) with simple structure and practical implementation for efficient control of complex control objects*.

3. QUANTUM ALGORITHM OF SELF-ORGANIZATION CONTROL

The generalized physical characteristic of self-organized systems which is of special interest for intelligent control systems design is the following characteristic: they possess *robust* and/or *flexible* structures. In this case, due to these properties the process of biologically reproducible self-organization includes the self-learning and self-adaptation processes.

The property of self-organization in natural and biologically reproducible systems is explained by a number of factors [22]. The first fact of tolerance is the *excessiveness* or the property of *distribution* of self-organization between the separated "protected" regions of evolution of the system structure in which the property of self-organization is satisfied. The second factor of internal robustness is its manifestation due to randomization, fluctuations, or noise. The third factor is the manifestation of stabilizing effects of flexible structures in feedback loops.

Let us consider the general characteristics of (biologically reproducible in the natural environment) self-organization processes which make the basis of design engineering (Fig. 1b), and are taken into account in development of the structures of intelligent control systems of the model of quantum algorithm of self-organization control for robust knowledge bases (Fig. 2b).

3.1. Biologically Reproducible Self-Organization Algorithm

Figure 6 illustrates the structure of the biologically reproducible self-organization algorithm developed in [11, 15, 18] based on the analysis of macro- and micro- models of self-organization.

A. The following information-representative models of the multilevel hierarchical structure of self-organization processes [11] were chosen as examples for analysis: (1) the behavior of the swarm of people in a tunnel [23]; (2) people, ant colonies, etc., in extreme conditions [23]; (3) auto traffic on highways [23]; (4) micro-level (ant colonies searching for food, intelligent active agents with information exchange, self-organization engineering of bacteria colonies) [24–26]; (5) quantum cooperation of insects [27], quantum self-organization of nanostructures (quantum corals) due to information transmission on the micro-level [28], and the change of the type of quantum correlation [29].

Note the common qualitative features of the models of self-organization processes. In the general form the self-organization process includes four mechanisms: (i) positive feedback; (ii) negative feedback; (iii) balanced relation between the used and potential resources of evolution of (information–thermodynamic) behavior of the dynamic system; (iv) multiple (physical and information) interaction between the components.

The natural (biologically reproducible) evolution of achieving self-organization of the dynamic system is based on the following stages (Fig. 6): (i) formation (determination) of the set of non-connected elements "building blocks" (templating) of the initial structure; (ii) self-assembling of the new structure; (iii) self-organization of the robust structure. In this case, as it was noted in Remark 6, the "self-assembling" in the general form differs from the "self-organization" by the absence in the "self-assembling" process of the preliminary choice of the type and form of correlation between non-connected elements, and the fact that the process is performed due to the local interactions and external (in the general case random) impacts [22–24]. These stages are shown in Fig. 6a.

Figure 6b demonstrates the presence of the real quantum self-organization in the quantum dot structure [28] due to the space–time correlation contained initially (coded) in the structure of "building blocks". This experimental fact [29] is the physical basis for construction of the mathematical model of the quantum algorithm of self-organization.

Below this information is applied for the particular example to the development of the mathematical model and the physical interpretation of the quantum self-organization algorithm (Fig. 6).

E x a m p l e 1. Physical interpretation and mathematical model of operators of quantum self-organization algorithms. Let us consider the general properties and features of the qualitative description of quantum effects in self-organization of evolution processes. The evolution of self-organization of pedestrians in different types of corridors [23, 30] will be considered.

Figure 7a shows the dynamics of motion of pedestrians in the limited space with different geometry and different types of cooperative behavior of people in the course of self-organization of the swarm of pedestrians.

Figure 7b shows the phenomenology of occurrence of different types of quantum correlation (temporal and spatial) which influences the form of self-organization in the case of information interaction of flows of pedestrians. In particular, the self-organization of counter flows of pedestrians (for avoiding collisions) in the case of application of superposition in the form of cooperation of pairs of people (block 1 in Fig. 7a) is shown. Similarly, the information exchange between separate pairs of pedestrians with entangled states for the counter flows of pedestrians in spaces geometrically separated by partitions (block 2 in Fig. 7a) is shown on the micro-level. Figure 7b also shows the role of quantum interference (block 3 in Fig. 7a) and the time correlation (block 4 in Fig. 7a).

Figure 8 shows the evolution of quantum operators describing the dynamic behavior of pedestrians in the situations shown in Figs. 7a and 7b.

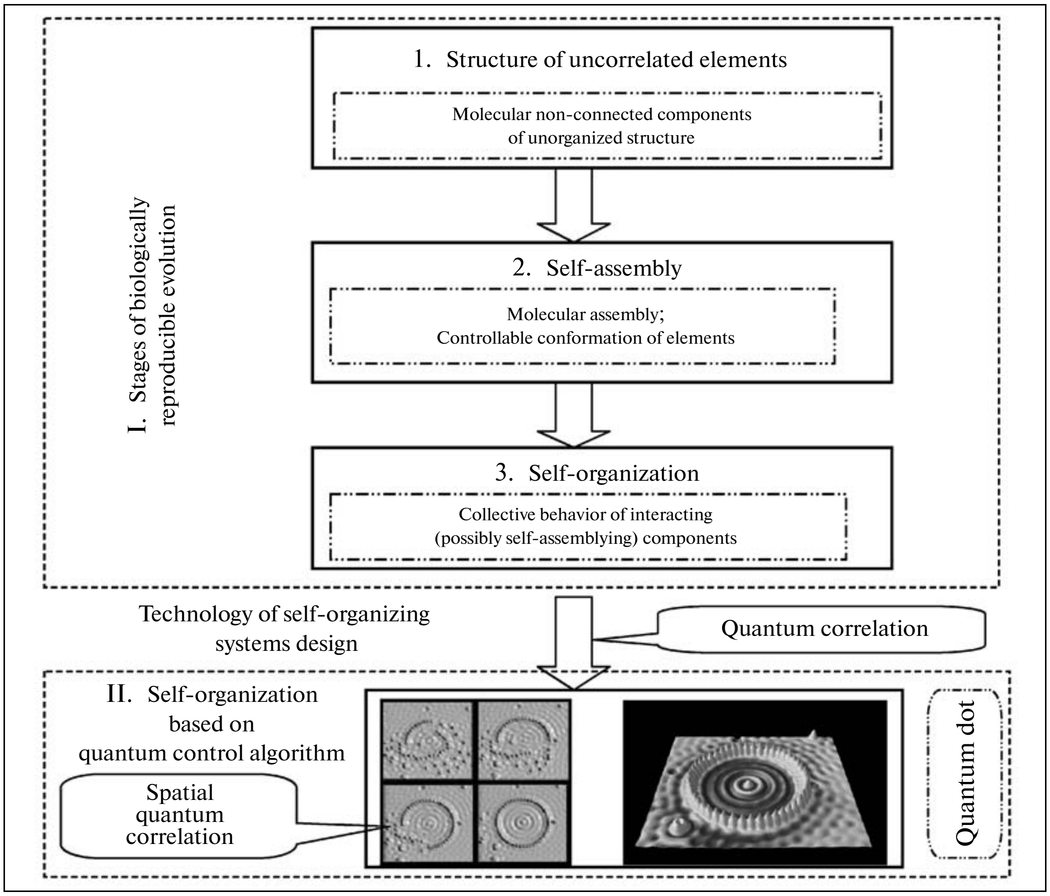
Thus, for example, the physical quantum superposition occurs in the case of motion of pedestrians in a tunnel (Fig. 8a). Figure 8b shows the evolution of quantum correlation (of entangled states) depending on the number of conflict collisions of pedestrians in the limited space [31]. Figure 8c shows the phenomenology of occurrence of quantum interference in the case of motion of pedestrians in counter flows in the case of complex geometry of flow separation (if pedestrians begin to panic [30]).

Thus, self-organization can be considered as the macroscopic process containing quantum (hidden) effects on the micro-level in the informationally interacting components [18, 32].

R e m a r k 7. There exists a "paradox" stating that self-organization contradicts the second thermodynamic law according to which the system evolution tends to chaos (entropy growth). This "paradox" was resolved [32] in terms of multiple interactions of macro- and micro-levels of evolution (open thermodynamic system) of self-organization and decreased entropy production on the macro-level (ordering growth) due to increased entropy production on the micro-level (behavioral chaos growth).

All physical examples of self-organization chosen for analysis possess the above mechanisms [11].

(a)



(b)

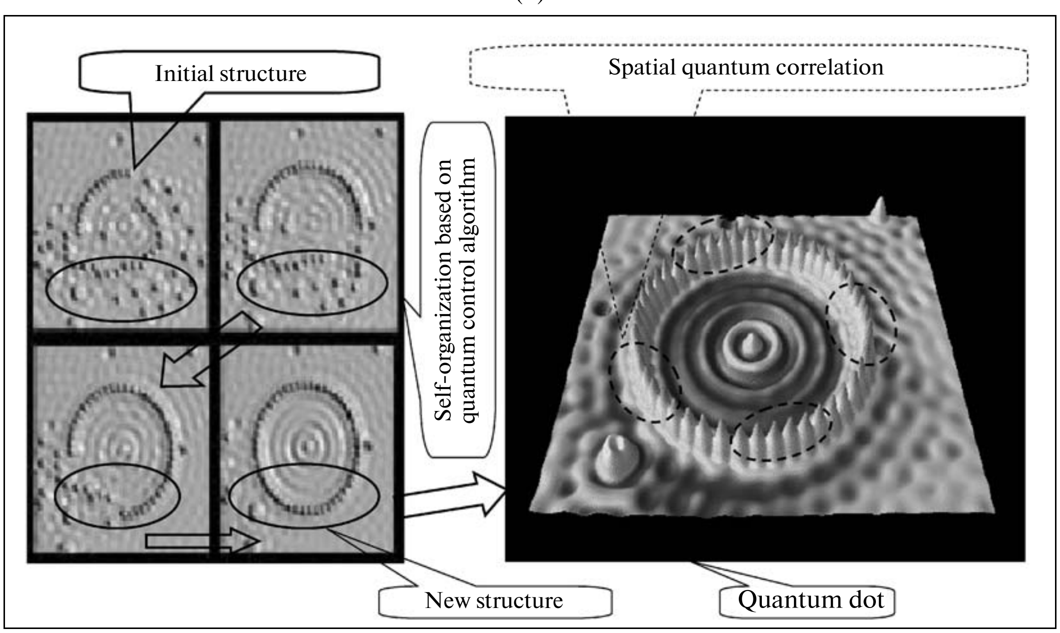


Fig. 6. Structure of biologically reproducible self-organization algorithm: (a) block diagram of quantum self-organization algorithm; (b) example of spatial–temporal quantum correlation in self-organization of quantum dot structure.

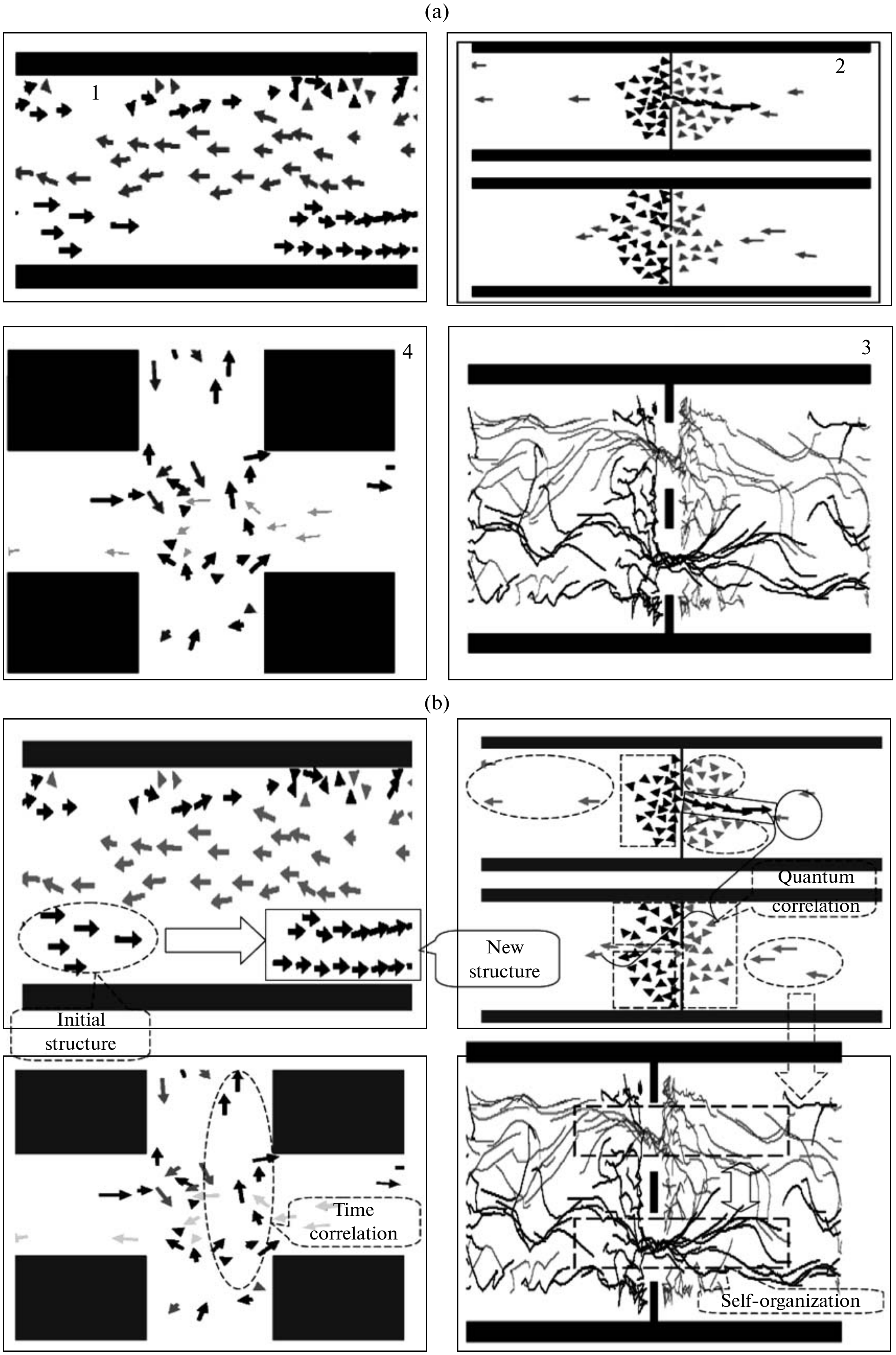


Fig. 7. Dynamics of motion of pedestrians in a bounded space with different geometry and different types of cooperative behavior of people: (a) examples of self-organization of a swarm of pedestrians; (b) phenomenology of appearance of different types of quantum correlation.

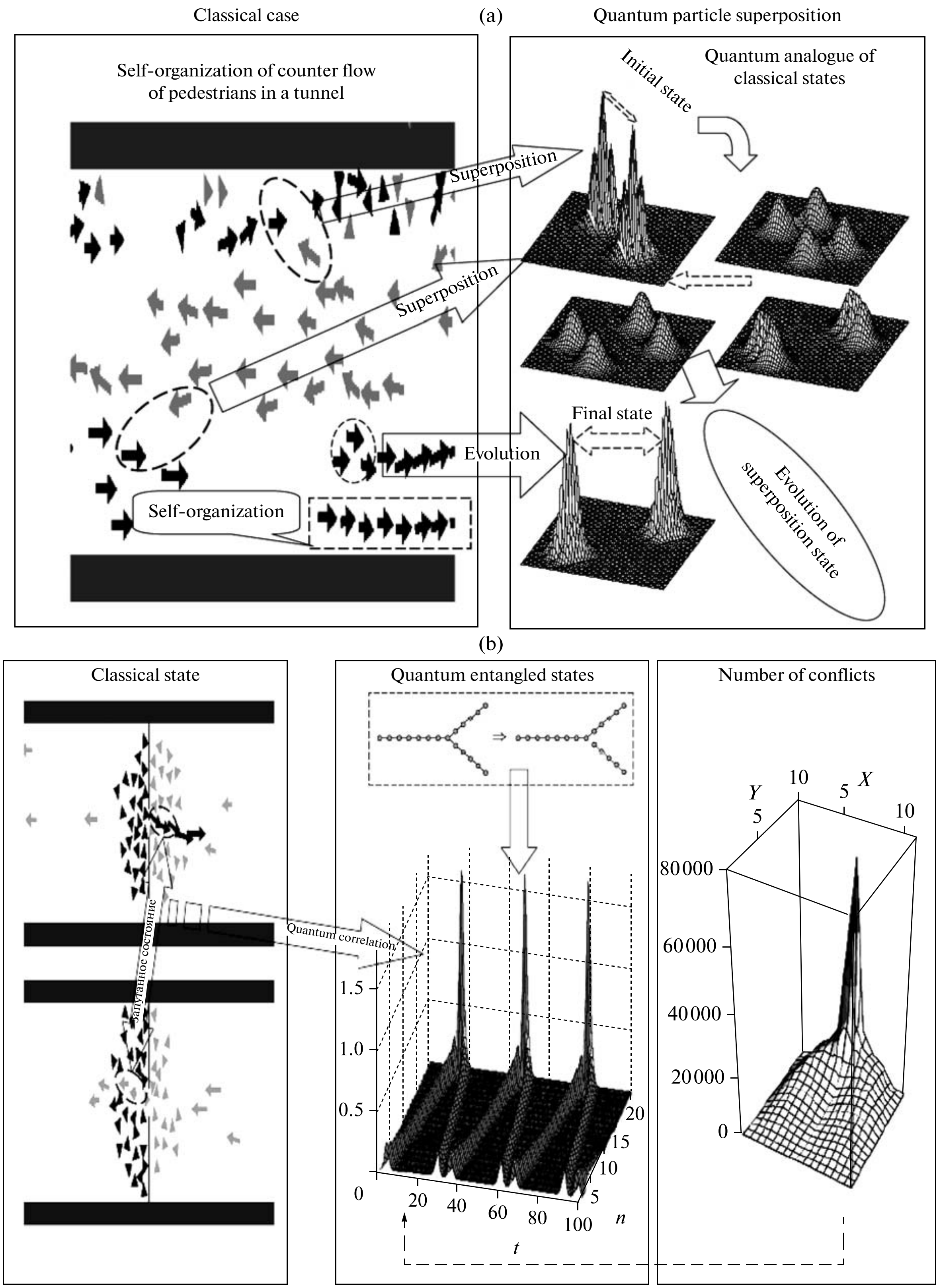


Fig. 8. Evolution of quantum operators describing the dynamic behavior of pedestrians: (a) quantum superposition; (b) evolution of quantum correlation (entangled states); (c) phenomenology of occurrence of quantum interference; QAC is the quantum algorithmic cell.

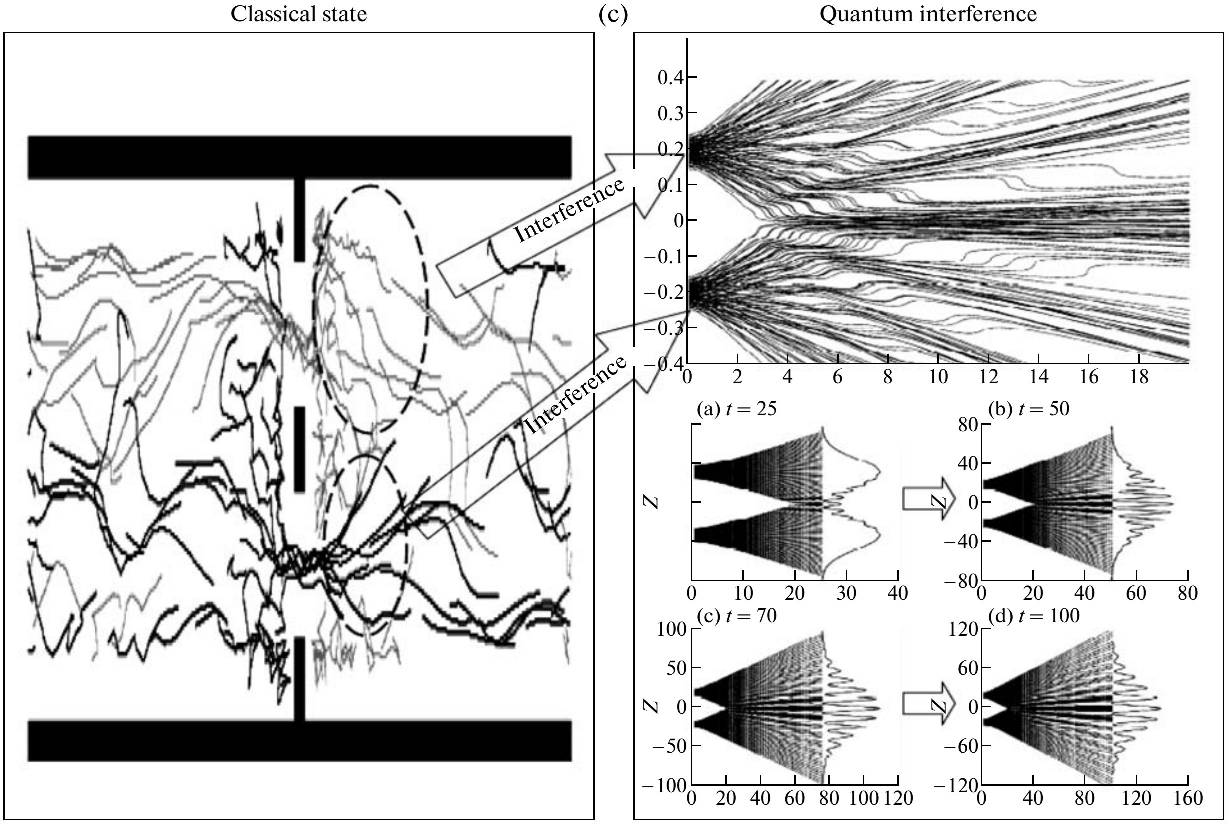


Fig. 8. (Contd.)

B. The analysis of the properties of the above models of self-organization testifies that they possess common biologically reproducible and experimentally detectable *quantum* effects in the self-organization processes, and the models are based on the following *information–thermodynamic* concepts: (i) the interaction between elements and micro- and macro-levels takes place with the information exchange. Thus, in the model of intelligent agents the micro-level is represented in the form of the information space in which the information exchange between agents takes place and results in the entropy reduction on the macro-level due to entropy growth on the micro-level [32–34]; (ii) the communication and information transmission on the micro-level (the phenomenon "quantum mirage" in quantum corals, see Fig. 6b) [28]; (iii) different types of quantum spin correlation (or entanglement) used in design of different self-organized structures (for example, in quantum dots [29]); (iv) the coordinated control due to information extraction and exchange (for example, self-organization of the team of robots [35] due to information–entropy exchange between agents [32] and organization levels [33]).

Figures 6–8 show the above algorithmic specific features of self-organization models.

3.2. Structure of Generalized Quantum Algorithm of Self-Organization Control

Let us consider the necessary definitions and facts of the theory of quantum computing, quantum information theory, and quantum algorithms [12–14] used in the structure of quantum algorithms of self-organization control [11].

A. *Structure and specific features of quantum algorithms.* Structurally the quantum algorithm is based on the main quantum operators of the theory of quantum computing: the *superposition* of classical states; the operator of formation of *entangled* states (or quantum oracle); the *interference* and (classically irreversible) *measurements*.

Figure 9 shows the qualitative comparison of the structures of quantum algorithms and the self-organization algorithm which yields the pictorial representation of the quantum nature of the self-organization algorithm (Fig. 6a).

First of all, let us note some specific features of the quantum algorithm.

Figure 9a shows the generalized structure of the quantum algorithm in the theory of quantum computing. According to the theory of quantum algorithms [13, 14] at the first stage the sought qualitative properties of the studied function are coded in the form of the

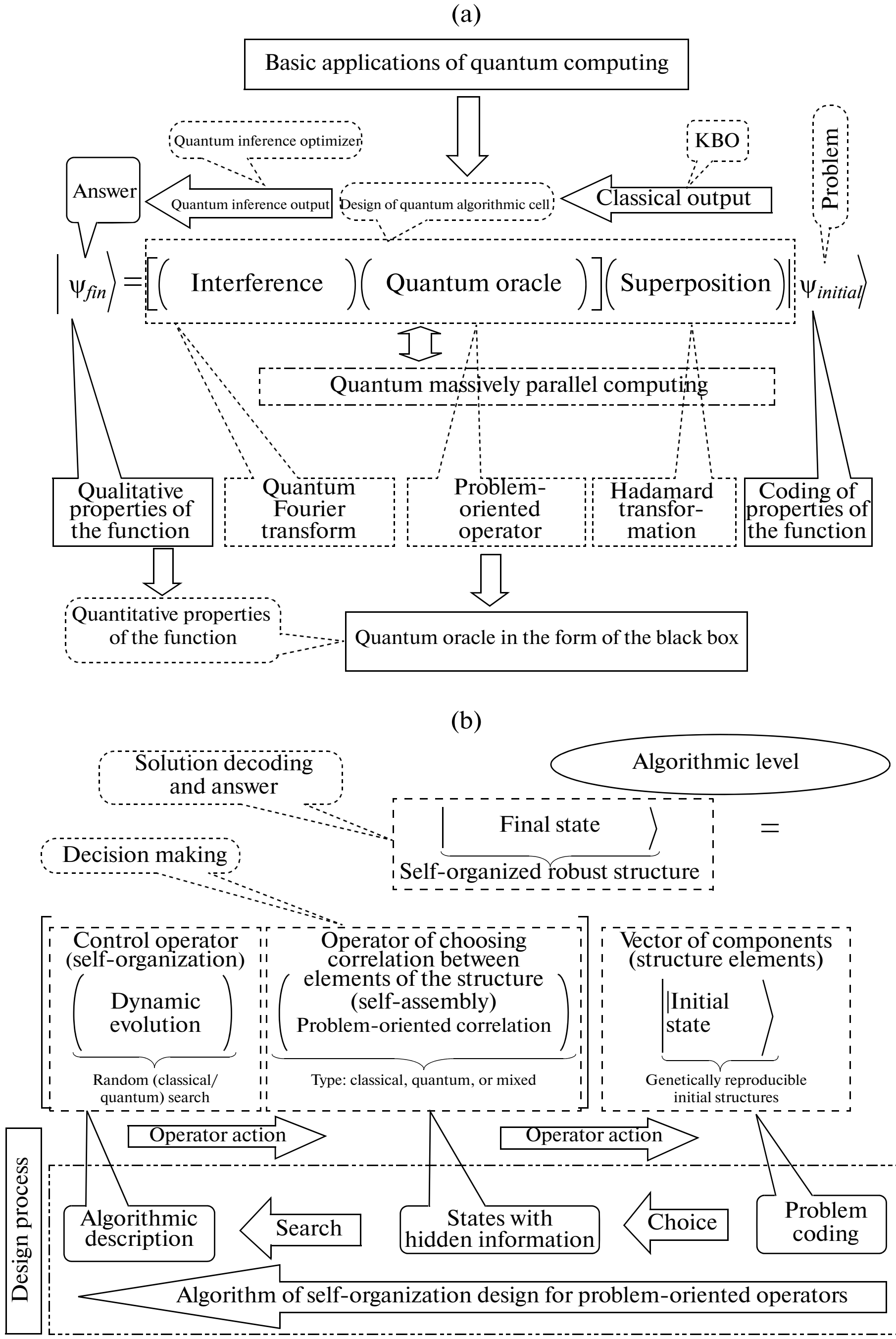


Fig. 9. Structure and main operators of quantum algorithms: (a) structure and functional connections of the quantum algorithm; (b) structure and functional connections of quantum algorithm of self-organization control.

wave function, and the oracle provides finding the sought properties. Therefore, Fig. 9a shows the quantum operator which performs this search. The form of the oracle operator determines the type of the quantum algorithm (search or decision making).

The design of the quantum algorithmic cell [21] allows one to include the description and action of different types of quantum computing on the classical computer. In the general form the model of quantum computing [11, 14] consists of five stages: (1) the preparation of the initial (classical or quantum) state $|\psi_{in}\rangle$; (2) the execution of the Hadamard transformation for the initial state for preparation of the state of superposition; (3) the application of the entanglement operator or the quantum correlation operator (quantum oracle) to the state of superposition; (4) the execution of the interference operator; (5) the application of the measurement operator for extraction of the results of quantum computing of $|\psi_{fin}\rangle$ and $|\psi_{fin}\rangle = H|\psi_{in}\rangle$, where H is the Hamiltonian of the system. The notation is given in Appendix.

Quantum algorithms make the physically grounded basis for the technique of acceleration of calculations (due to the application of massively parallel computing) and for the search of solutions to complex problems.

In this case such quantum laws as the *superposition* for extending the space of possible solutions, *quantum massive parallelism* of calculations for acceleration of the search of solutions, and *constructive quantum interference* for extracting the sought solution [12, 13] are efficiently used.

Additionally to these resources the *quantum correlation* is considered as the new physical calculational resource which provides sharp increase in the successful search of solutions to algorithmically unsolvable problems [9] which were not used in classical computing earlier. This formalism can be expressed in terms of quantum states or operator transformations; however, we are also interested in the possibility of adequate description of quantum states and effects in terms of logical inference: application of the conventional formalism, its power and expressiveness as the *quantum system of logical fuzzy inference* [10, 36].

The operation of quantum operators is denoted by corresponding notes in Fig. 9a; they work in the iterative regime depending on the type of the quantum algorithm.

The description of the main quantum operators is given in Appendix. Now let us consider the specific features of quantum algorithms of self-organization control shown in Fig. 9b.

B. *Generalized structure of quantum algorithm of self-organization control.* From the point of view of the theory of quantum computing, the quantum algorithm of self-organization control includes all necessary operators (operators act from left to right) and contains the following stages and specific features (Fig. 9b): (1) the preparation of the state of superposition; (2) the choice of the type of quantum correlation; (3) the application of the quantum oracle operator (model of the "black box"); (4) the transmission of extracted information (dynamic evolution of the "intelligent" state of the control signal for the criterion of minimal information entropy); (5) the quantum correlation is higher than the classical correlation and is considered as the additional resource of the quantum algorithm; (6) the application of the operator of constructive interference for extracting the sought (found) solution; (7) the effect of massively parallel quantum computing; (8) the amplification of the probability amplitude of the sought solution and the solution of the classically algorithmically unsolvable problem based on the application of efficient quantum decision making strategies [11].

R e m a r k 8. For more pictorial perception of the results of comparison in Fig. 9b the operation of quantum operators (similar to Fig. 9a) and designing of the effect of self-organization by problem-oriented operators are marked by notes. Quantum operators are applied in the iterative regime depending on the type of the quantum algorithm. These effects are achieved using quantum operators whose basic properties are given in Appendix.

R e m a r k 9. The choice and determination (of the type and form) of quantum correlation depends on the class of nonlinearity of the control object. The problems of calculation of the "intelligent" state of the control signal for the criterion of minimal Shannon information entropy are considered in Appendix.

C. *Specific features of mathematical model of quantum algorithms of knowledge self-organization control.* The mathematical model of knowledge self-organization (Fig. 9b) is described in the following quantum algorithm:

$$\underbrace{|\text{Final state (control laws)}\rangle}_{\text{Self-organization of robust structure of the knowledge base}}$$
$$= \left[\underbrace{(\text{Process evolution})}_{\text{Random quantum search}} \cdot \begin{pmatrix}\text{Quantum}\\ \text{computing}\end{pmatrix} \cdot \overbrace{\underbrace{(\text{Correlation type})}_{\text{Classical, quantum, mixed}}}^{\text{Problem orientation}}\right] \left.\overbrace{\underbrace{|\text{Initial state}\rangle}_{\text{Bio-inspired states}}}^{\text{"Building" blocks}}\right\} \cdot \begin{pmatrix}\text{Reproduced by knowledge base}\\ \text{optimizer with soft computing}\end{pmatrix} \quad (3.1)$$

In (3.1) the notation $(\cdot)$ means the application of the corresponding tool or operation.

At the first stage of algorithm (3.1) the knowledge base optimizer with soft computing is used to create the "building" blocks of knowledge self-organization based on the genetic algorithm in the form of control laws for the gains of the fuzzy PID controller on the basis of production rules of the knowledge base. The

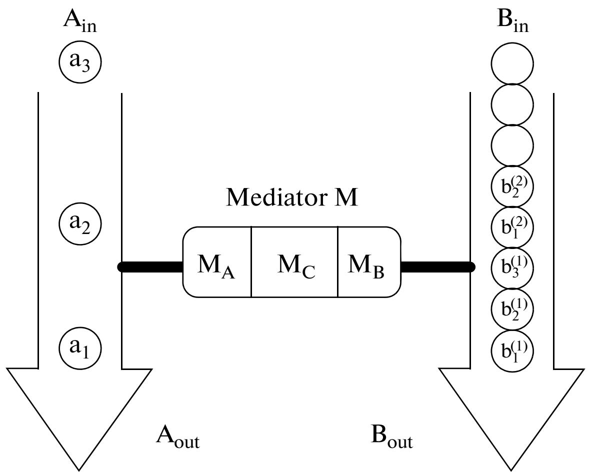


Fig. 10. Information exchange process via quantum channels.

obtained trajectories of the control law are considered as the classical states, the chaotic paths of the intelligent collective motion of particles in the "swarm" with information interaction (swarm intelligence) which is one of the efficient methods of algorithmization of quantum models of description of the collective particle behavior (swarm method of collective particle behavior) [18].

Under certain assumptions the swarm method has the calculational complexity of the simulation algorithm [11] of the dynamics of collective particle interaction with linear dependence on the number of particles, which yields great advantages in the case of constraints on the memory and time of simulation.

Let us consider (without losing generality) the simple case of two trajectories of the control laws of gains; these trajectories belong to the set S_1 and S_2. According to (3.1), the superposition of the classical states on the set $S = S_1 \bigcap S_2$ forms the unified quantum state in the form

$$|\psi_S\rangle = \sum_j \lambda_j |\psi^j_{S_1}\rangle \otimes |\psi^j_{S_2}\rangle, \qquad (3.2)$$

where $|\psi^1_{S_1}\rangle, |\psi^2_{S_1}\rangle, \ldots$ and $|\psi^1_{S_2}\rangle, |\psi^2_{S_2}\rangle$ form the orthonormal calculational basis in the space of states S_1 and S_2, respectively. Physically (3.2) is considered as one state in which entangled states are formed from the process of particle motion along classical trajectories due to the information exchange.

Figure 10 shows the process of information exchange via quantum connection channels [37] between particles on the trajectories $A_{\text{in}} \in S_1$ and $B_{\text{in}} \in S_2$.

Here, the mediator M is the compound quantum object with the finite memory dimension d_M uniting three subsystems M_A, M_B, and M_C using the Hamiltonian H. The mediator M is the efficient channel which connects two different parts, the source of message **A** and the receiver of information **B**. In this case, **A** is supported by the quantum register A and B, respectively. The quantum register possesses partially ordered memory $a_1, a_2, \ldots$. The source **A** sends the message to **B** which is stored in the quantum memory $a_1, a_2, \ldots, a_n$, connected with the subsystem M_A of the mediator M; each memory element contacts M_A once, following the order indicated in Fig. 10 (shown by the arrow, i.e., first a_1, then a_2, etc.). The receiver **B**, having received the message from **A**, forms the memory b using the corresponding state $|v\rangle$ and is united with the subsystem M_B of the mediator M, following the order shown in Fig. 10; in this figure, A_{in} and B_{in} mean the input ports of devices used by **A** and **B** for transmission of q-bits at the contact with M. Similarly, A_{out} and B_{out} mean the output ports where q-bits are formed after the contact with the mediator M.

It was shown in [37] that the state $|v\rangle$ of such quantum connection channel forms the unified quantum state with the mediator, and the particular spur of the density matrix $|v\rangle\langle v|$ (together with the density matrices of the source **A** and the receiver **B**) is superposition (3.2) in the form of Schmidt decomposition in the calculational basis $\{|0\rangle, |1\rangle\}$. In this case in the quantum information theory [11] quantum connection channels are used to transmit superposition of signals (3.2) with preservation of different types of correlation between agents.

In the theory of quantum computing (see Appendix) the process of calculation begins with the action of the evolution operator U_f on the "initial state" $|00\ldots0\rangle$ in the form $U_f = \otimes U_{f(i)}$, the unitary generalized Walsh–Hadamard transformation, where

$$U_{f(i)} = \begin{pmatrix} \sqrt{f(i)} & -\sqrt{1-f(i)} \\ \sqrt{1-f(i)} & \sqrt{f(i)} \end{pmatrix}$$

and $\sqrt{f(i)}$ determines the probability amplitude of the ith classical state in the quantum superposition.

As a result for (3.2) we have $U_f|00\ldots0\rangle = |s_f\rangle$, where $|s_f\rangle$ is the state of superposition of the finite number of classical states. Thus, each operator $U_{f(i)}$ maps the separate quantum bit of the initial state into the mixed state of superposition with the given probability of the state $f(i)$. The geometrical interpretation of the operator $U_{f(i)}$ is the Bloch sphere with rotation about the axis y by the angle $\theta_i = 2\arcsin\left(\sqrt{f(i)}\right)$. Walsh–Hadamard transformation is used to form the hidden (unobserved) mixed correlation in the superposition of signals of two classical knowledge bases of the fuzzy controller. Entangled states (quantum correlation) are simulated by the quantum oracle which can determine the maximal probability amplitude on the set of corresponding classical superposition states.

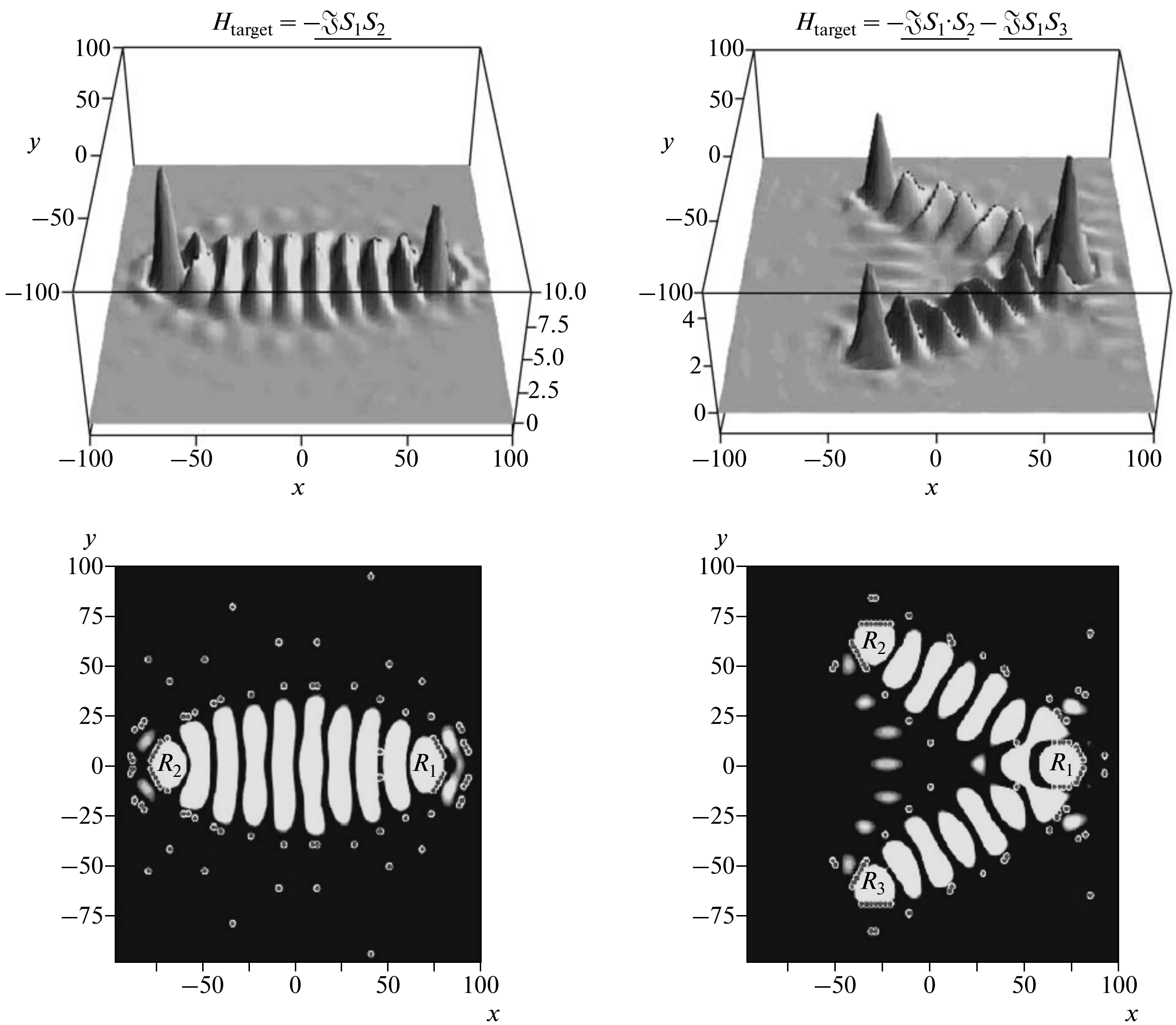


Fig. 11. Example of the influence of the type of quantum correlation between spins on the form of the nanostructure of the quantum "coral".

The description of the physical basis of formation of the quantum correlation, its role in the mechanism of formation of self-organization, and the influence on the form of the structure, as well as the introduction of the physical interpretation of the mathematical model of the corresponding quantum algorithm in this contingency control situation were not included in problem [10] and therefore, were not completely analyzed on the meaningful physical level.

Figure 11 shows the influence of the type of quantum correlation between the spins on the form of the nanostructure of the quantum "coral" [29].

The change of the type of quantum correlation from ($\mathcal{J}S_1S_2$) to ($\mathcal{J}S_1S_2 + \mathcal{J}S_1S_3$) in the target Hamiltonian H_{target} by the interconnected spin chain results in the considerable change of the type of the self-organized structure of the quantum coral.

Thus, the existence and the influence of the type of quantum correlation on the formation of the self-organized structure are the experimentally established facts which are taken into account in the block "Correlation type" of quantum algorithm (3.1) based on the model of information exchange vial quantum connection channels (Fig. 10). The interference operator (in the form of the identity operator) together with the classical measurement (observation) procedure are used to extract the correlation state from the "intelligent" state (see Fig. 12b) for the maximum of the probability amplitude (minimum of Shannon information entropy).

Note some specific features of the physical interpretation of the quantum algorithm of self-organization control shown in Figs. 6a and 9b.

D. *Physical interpretation of quantum algorithm of self-organization control.* From the point of view of the

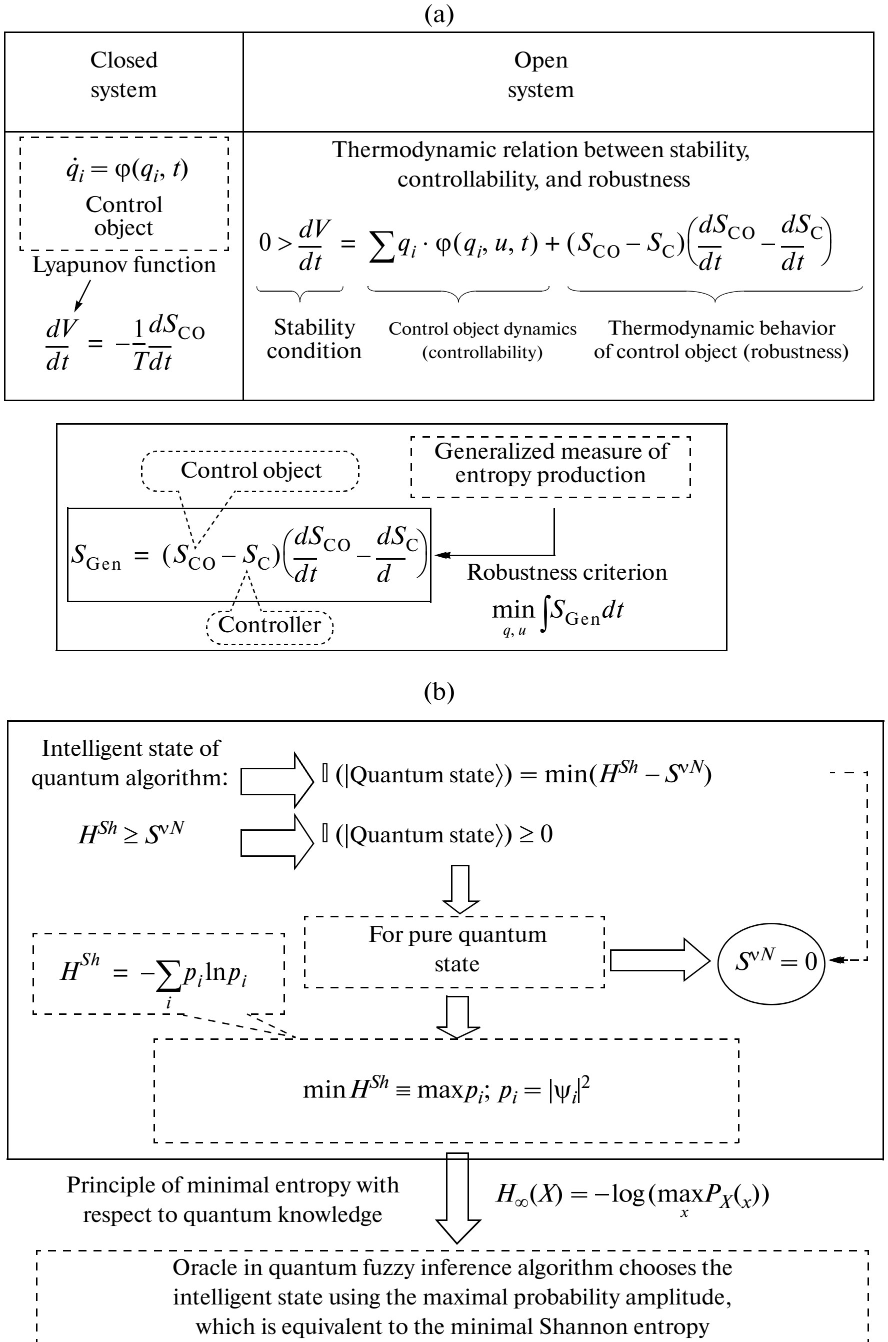


Fig. 12. Robust control quality design: (a) thermodynamic quality criterion of robust control; (b) choice of "intelligent" quantum state.

process of biologically reproducible evolution of self-organization (Figs. 6a, 9b) at the first stage of application of the quantum algorithm (3.1) the superposition operator implements the procedure of formation of the set of non-connected elements of the structure in the form of "building blocks" (templating) and codes the sought solution (see the corresponding note "Problem coding" in Fig. 9b). In this case, the procedure of interaction of elements of micro- and macro-levels with information exchange between active agents is formed.

Remark 10. It should be mentioned that here, following the ideas and definitions of the self-organization theory [22, 23] the active agents of the macro-level [32, 34] are the current values of the control process obtained as a result of reaction of robust knowledge bases of the fuzzy controller. Active agents of the micro-level here are the virtual values of the control process obtained as a result of application of the quantum principle of complementarity to real values of the control state on the macro-level.

The choice of the type of quantum correlation realizes the self-assembling of the required structure using the resource of interaction via communication and information transmission on the micro-level [32–34]. In this case, the type of correlation determines the level of robustness of the intelligent control system. In this application of the quantum oracle, the "intelligent" quantum state of the self-organization structure, which contains valuable information for application and implementation of coordinated control, is calculated. The interference is used for extracting the results of coordinated control and robust knowledge design.

The particular model of the quantum algorithm of self-organization control based on the quantum fuzzy inference and its application in the technology of robust knowledge base design was considered in [18]. In particular, the following example was given: the quantum self-organization algorithm (based on the self-organization of the behavior of the swarm of people in the tunnel) for (real-time) formation of knowledge based on robust knowledge bases in fuzzy PID controllers under the conditions of uncertainty of initial information and contingency control situations.

E. *Goal of application of quantum algorithm of self-organization control.* Figure 12a shows the main thermodynamic relation of the quality of robust intelligent control and the optimization criterion used in the quantum self-organization algorithm [20].

R e m a r k 11. In Fig. 12 the following notation is used: V is the Lyapunov function; S_{OY}, S_P is the entropy production in the control object and the controller, respectively;

$$V = \frac{1}{2}\sum_{i=1}^{n} q_i^2 + \frac{1}{2}S^2; \quad S = S_{OY} - S_P; \quad \dot{q}_i = \varphi(q_i, u, t)$$

are the motion equations of the control object; S^{vN} is the von Neumann quantum entropy; ψ is the wave function; and p_i is the event probability. The derivation of relations in Fig. 12a can be found in [2, 8].

Figure 12b illustrates the details of the calculational process of the choice of "intelligent" quantum state (see Appendix, Example 6) for the principle of minimal Shannon information entropy (maximal probability amplitude). It was already noted in Section 3.1 that in the course of evolution of self-organization the balanced relation between the used and potential resources of evolution of the (information–thermodynamic) behavior of the dynamic system is achieved [11].

Note the following aspects of development of the self-organization model: (1) the goal of the developed quantum algorithm of knowledge self-organization control is achieving the thermodynamic criterion (see Fig. 12a) of the optimal relation between the stability, controllability, and robustness which is used in real time in the control loop [16, 19]; (2) the principle of minimal generalized entropy of the "dynamic control object + controller" system provides simultaneous achieving the global robustness and implementing the optimal intelligent control with minimal loss of the useful energy resource [8, 20].

F. *Difference of the model of quantum control algorithm of self-organization from biologically reproducible evolution of self-organization.* The main differences are: (1) the quantum algorithm of self-organization control is described as the logical process of application of valuable quantum information extracted from classical states using quantum decision making strategies and facts of quantum information theory; (2) it contains the choice of the type and form of quantum correlation which influence the formation and form of the structure of the designed system; (3) structurally the quantum algorithm includes all necessary qualitative specific features and operators of natural (biologically reproducible) self-organization, which are described by quantum operators of the theory of quantum computing; (4) it is a new search quantum algorithm, which can be used to solve classically algorithmically unsolvable control problems; (5) it is implemented in real time using the reaction of classical fuzzy controllers on the new control error in the contingency control situation for the robust intelligent control design; and (6) it supports optimal thermodynamic relation between the stability, controllability, and robustness for real-time design of intelligent self-organized control processes.

Therefore, the quantum algorithm of knowledge self-organization control (3.1) contains the physically grounded and experimentally established quantum operators and is related to the new class of search quantum algorithms depending on the choice of the type and form of the problem orientation of quantum correlation.

4. STRUCTURE OF SELF-ORGANIZED INTELLIGENT CONTROL SYSTEM AND DECOMPOSITION OF THE VECTOR OPTIMIZATION PROBLEM

The structure of the self-organized intelligent control system based on models of knowledge base optimizer [7, 38–40] and quantum fuzzy controller [10, 16, 19, 20] is shown in Fig. 13.

In Fig. 13 notes show the main contingency control situations and the functional operations supporting self-organization in real time.

It was already noted in Remark 5 that the self-organization of robust knowledge bases is the possibility of formation of robust control signals for parameters (gains) of actuators forming the control force of the control object based on the reaction of production rules of robust knowledge bases of fuzzy PID controllers designed earlier on the control error under the

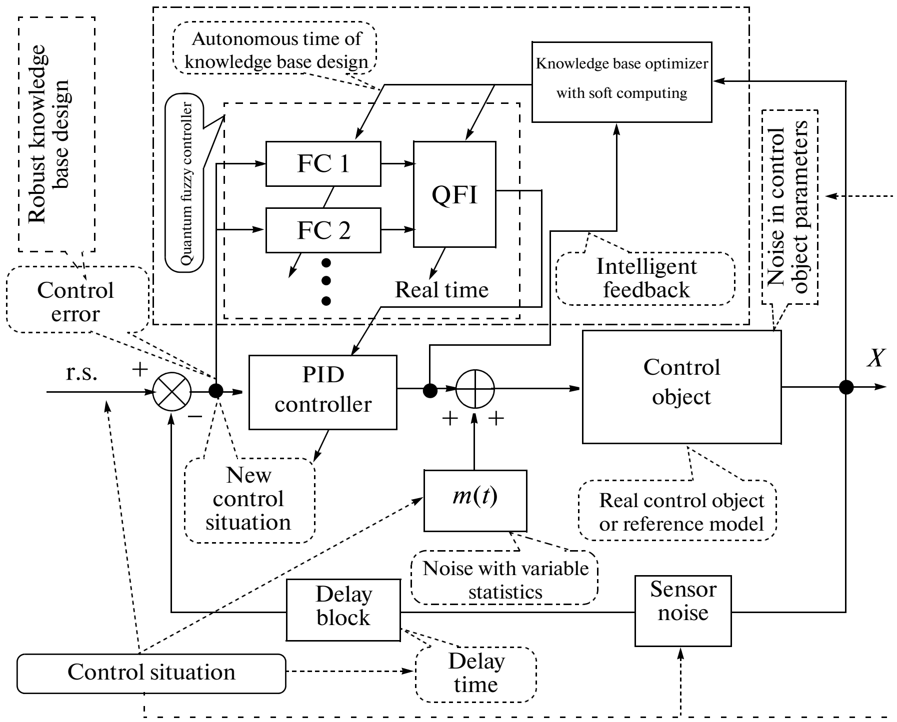


Fig. 13. Structure of intelligent control system with knowledge base optimizer with soft and quantum computing.

conditions of uncertainty of new initial information or contingency control situations.

The combination of the finite number of knowledge bases designed earlier and the model of the quantum fuzzy controller into one block "Quantum fuzzy controller" provides the possibility of forming in real time the robust control at the output of the PID controller using the reactions of production rules of these knowledge bases on the contingency control situation. In this case it is sufficient to use just minimal information on the change of the control situation contained in the new control error [38], which is shown in Fig. 13.

This is an advantage of the applied principle of compensation of the control error based on the global negative feedforward and the introduced intelligent feedback. Therefore, the information on the reaction of the control object on the contingency control situation contained in the new control error is sufficient for realization of self-organization of the hybrid fuzzy PID controller.

Remark 12. In Fig. 13 the control signal for the gains of the PID controller is calculated in real time. The signal of the control force is corrected via the fitness function in the knowledge base optimizer using the intelligent feedback. These operations are shown in Fig. 13 by "piercing" signals of corresponding functional blocks. This figure also shows the possible contingency control situations and their combinations: noise can be delayed in the measurement channel; the block "Sensor noise" can be situated after the delay time, and so on. The particular control situation and the schematic diagram of automatic control system determine the particular position of blocks.

Note that in the information technology (Fig. 1b) the finite set of knowledge bases for given control situations is designed (the regime of learning without quantum fuzzy controller) off-line based on the knowledge base optimizer with soft computing (Fig. 2a). Then the model of quantum fuzzy controller is used on-line (Fig. 2b) and the process of self-organization of the knowledge base is implemented using the reactions of fuzzy controller on the contingency control situation (see Section 4.2, example of application of the quantum algorithm of self-organization control for intelligent control system for the solution of the multicriteria optimization problem).

4.1. Specific Features of Design of Knowledge Base Based on Knowledge Base Optimizer

For more complete perception of the results of simulation and the fuzzy controller design we note some specific features of software support of knowledge base optimizer (described preliminarily in Section 1.1) and the technical characteristics of knowledge bases for fuzzy controller designed based on it.

Figure 14 shows the general structure of simulation of the main blocks of the fuzzy controller and the structure of the main blocks of the software support of knowledge base design.

Figure 14 also shows the example of the block of fuzzy inference built in the knowledge base optimizer

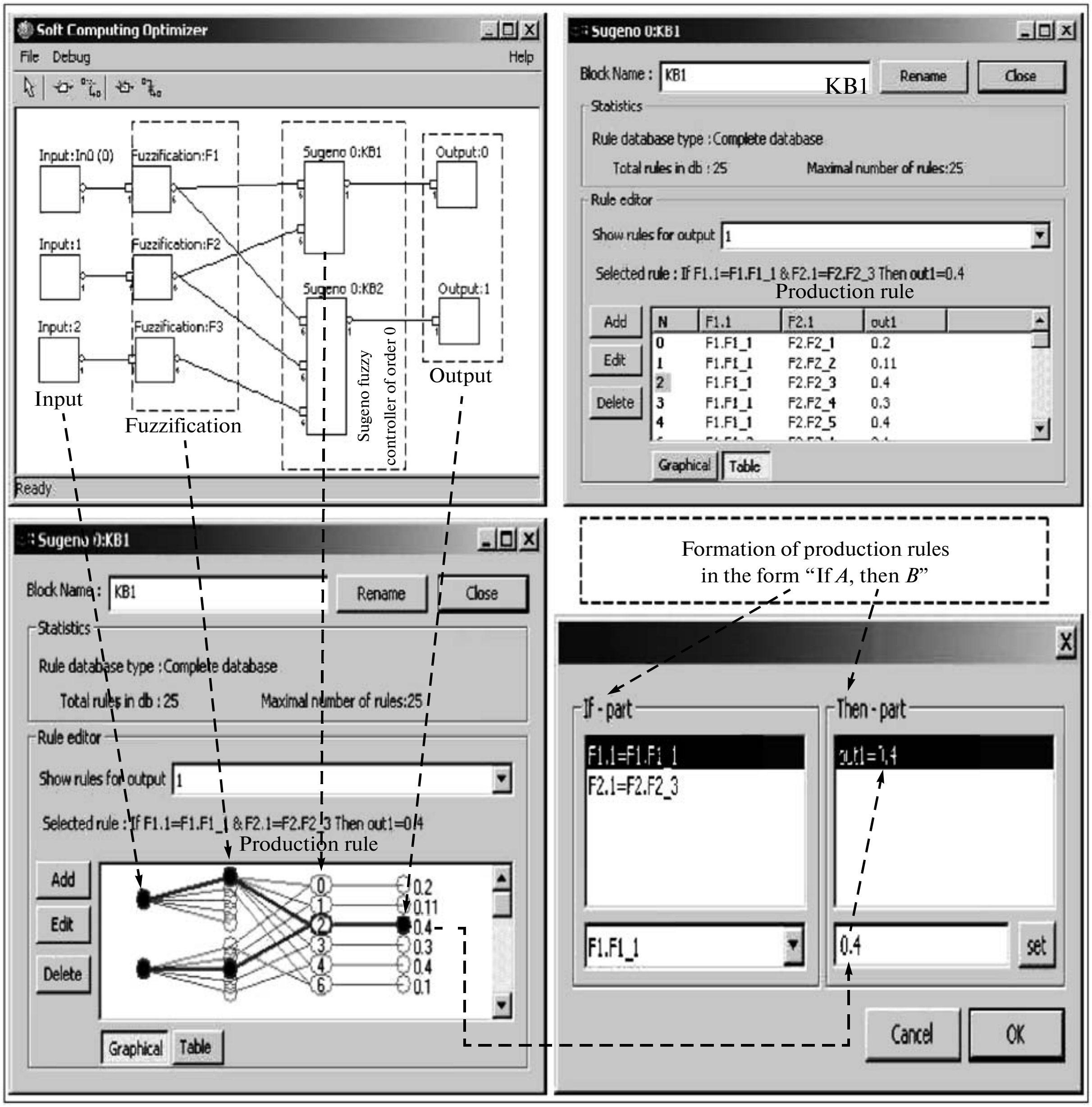


Fig. 14. Structure of simulation of fuzzy controller and software support of knowledge base design.

using the Sugeno fuzzy controller (zero order) model and the form of the production rule. Structurally the knowledge base optimizer has the program interface compatible with MathLab and is software compatible with the block of quantum fuzzy controller via the built-in interface (see [7, 10] for detailed description).

Let us consider briefly designing the learning signal for the optimal control (approximation of the learning signal with the given error) as the output signal of the knowledge base in the fuzzy controller based on the structural diagram shown in Fig. 2a. We take the learning situation **S1** for the random Gaussian noise (Fig. 15) as the learning situation.

The quality criterion of the control is determined in the form of the minimal control error and is considered as the fitness function of the genetic algorithm with the space of search of amplification coefficient (0, 5). For designing production rules of the knowledge base in the given control situation (see Fig. 14) the following is determined: (1) three input variables $\left\{e, \dot{e}, \int edt\right\}$ describing the dynamic behavior of the control error and (2) three output variables $\{k_p, k_d, k_i\}$ representing the gains of the designed fuzzy PID controller. In the knowledge base optimizer the process of knowledge base design yielded the following characteristics: (1) the number of membership functions for

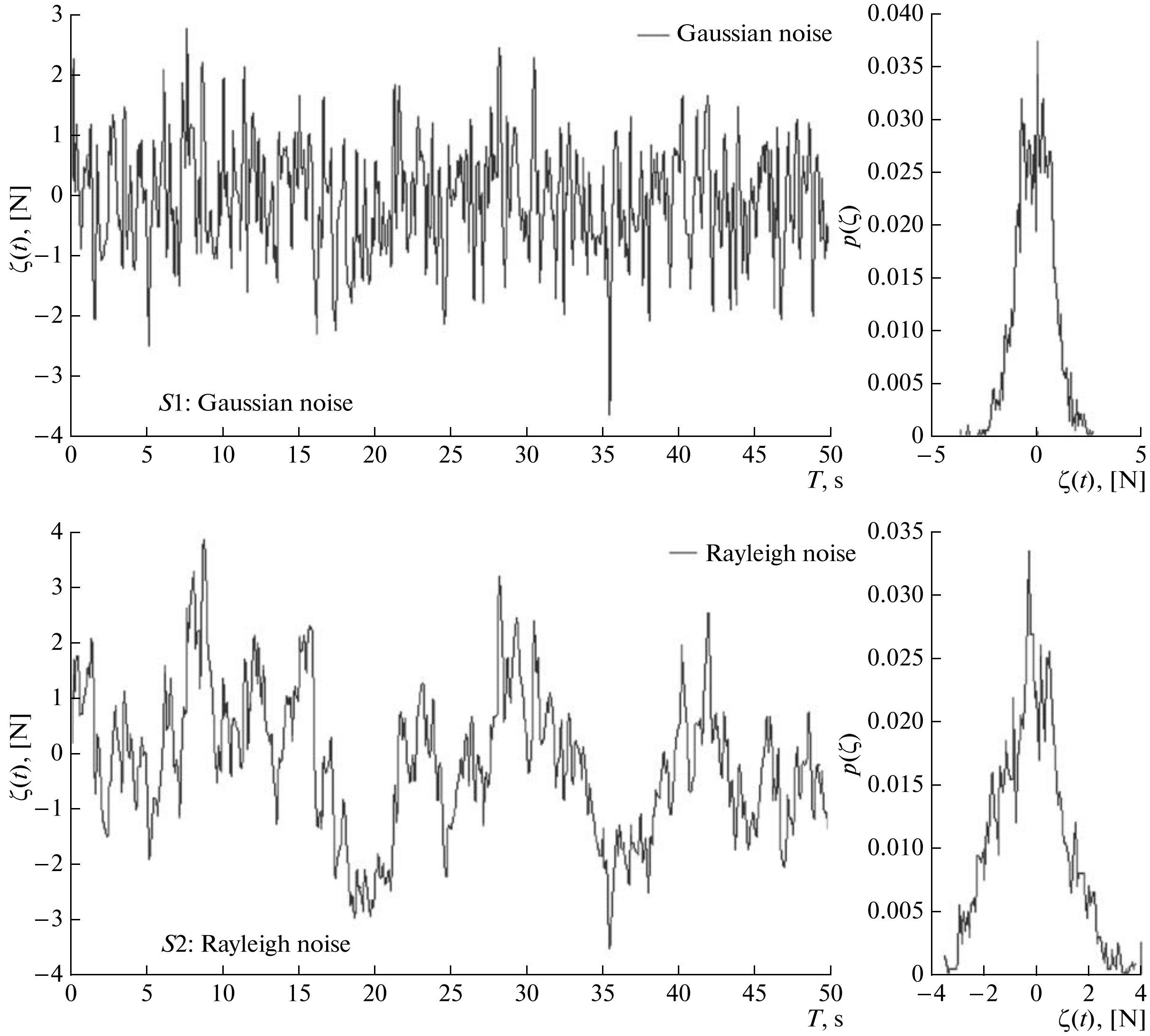


Fig. 15. Form of random noise used in learning situations (**S1**, **S2**).

each output logical variable (7, 9, 9) (optimal determined by genetic algorithm 1), respectively; (2) the total number of production fuzzy rules $7 \times 9 \times 9 = 567$; (3) the optimal number of production rules chosen for the knowledge base using the criterion of frequency of requesting the knowledge base was 25; and (4) the optimal number of production rules chosen by genetic algorithm 2 was 25.

Figure 16 shows an example of the optimal form of the membership function for the third output variable k_I (the gain of the integral error) and the result of approximation (denoted by the dark line) according to the scheme in Fig. 3 which eliminates excessive information in the learning signal.

For comparison we present the results of operation with fuzzy neural networks of AFM (Adaptive Fuzzy Module, developed by ST Microelectronics) type. The number of manually determined membership functions for each linguistic variable was five; the total number of production fuzzy rules was 125 and the number of activated rules was also 125. In AFM the number and form of the membership functions are determined by the expert, while in the knowledge base optimizer all design operations are automated in the optimal way based on the genetic algorithm.

The analysis of the results demonstrated that the structure of the knowledge base with the indicated parameters designed based on the knowledge base optimizer provides higher robustness of the fuzzy controller, as compared with the fuzzy neural network (125 rules) and the standard PID controller with constant parameters (5, 5, 5) [7].

The structural diagram in Fig. 13 can efficiently be used for solution of a complex problem, such as the decomposition of multicriteria optimal control prob-

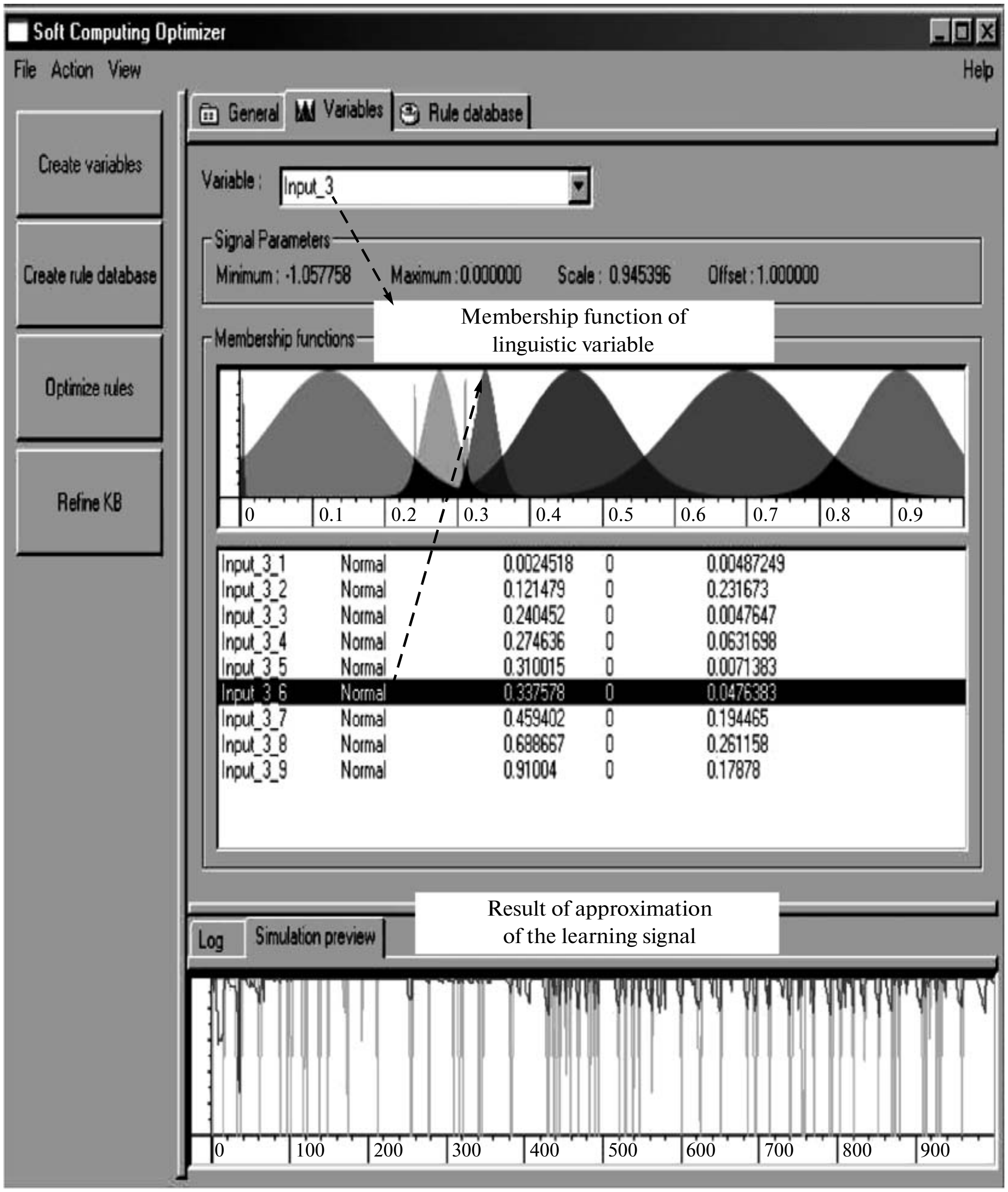


Fig. 16. Example of the optimal form of membership function for the third output variable k_I (amplification coefficient of integral error).

lem (with a vector cost function) under the conditions of uncertainty of control situations.

4.2. Decomposition of Multicriteria Optimal Control Problems

In this case the knowledge base of the individual fuzzy controller is designed using the knowledge base optimizer with the separate criterion (for example, minimal control error, minimal absolute control error, time of transient process, time and amplitude of overcontrol, minimal generalized entropy production, etc.) for the fixed control situation. These control quality criteria are identified with the fitness function in the corresponding genetic algorithm in the knowledge base optimizer.

The schematic diagram in Fig. 13 is used to determine the reaction of individual fuzzy controllers on the contingency control situation; the quantum fuzzy controller is used to aggregate the cost function and form the robust multicriteria optimal control signal for gains of the fuzzy PID controller. In this case the "intelligent" state chosen using the criterion of minimal entropy includes all characteristics of the multi-

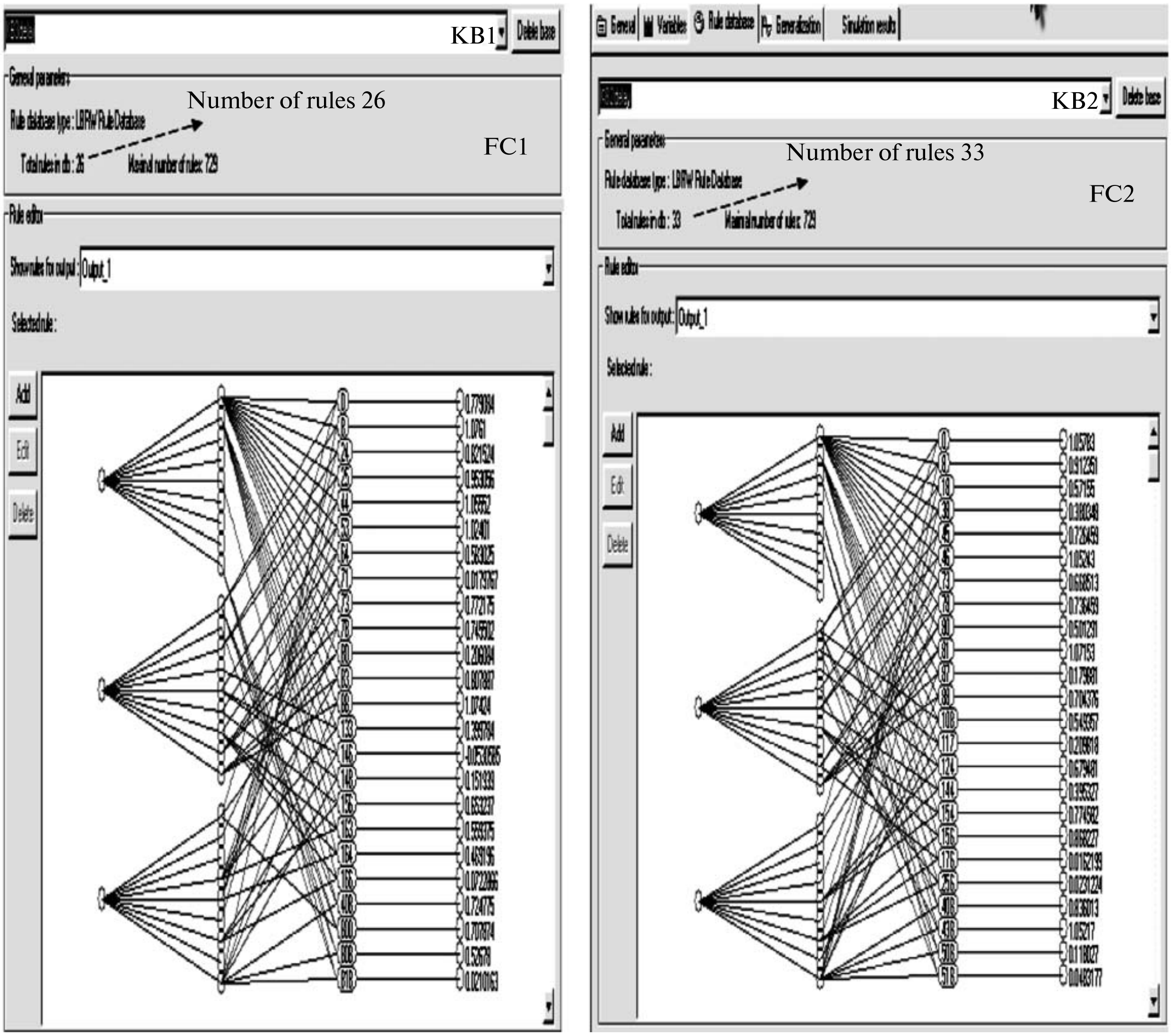


Fig. 17. Form of Knowledge bases 1 and 2 with corresponding activated production rules.

criteria control best for this situation; the priority between the control criteria is established automatically, and the dominating component is established.

E x a m p l e 2. The solution of the vector optimization problem based on the decomposition of the knowledge base. Let us consider the dynamic model of the system "moving cart + overturned pendulum" controlled by the fuzzy PID controller according to the structural diagram in Fig. 13.

The motion of the dynamic system "reversed pendulum–moving cart" is described by the following equations [41]:

$$\ddot{\theta} = \frac{g\sin\theta + \cos\theta\left(\dfrac{u + \xi(t) + a_1\dot{z} + a_3 z - ml\dot{\theta}^2\sin\theta}{m_c + m}\right) - k\dot{\theta}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2\theta}{m_c + m}\right)},$$
$$\ddot{z} = \frac{u + \xi(t) - a_1\dot{z} - a_2 z + ml(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta)}{m_c + m}, \quad (4.1)$$

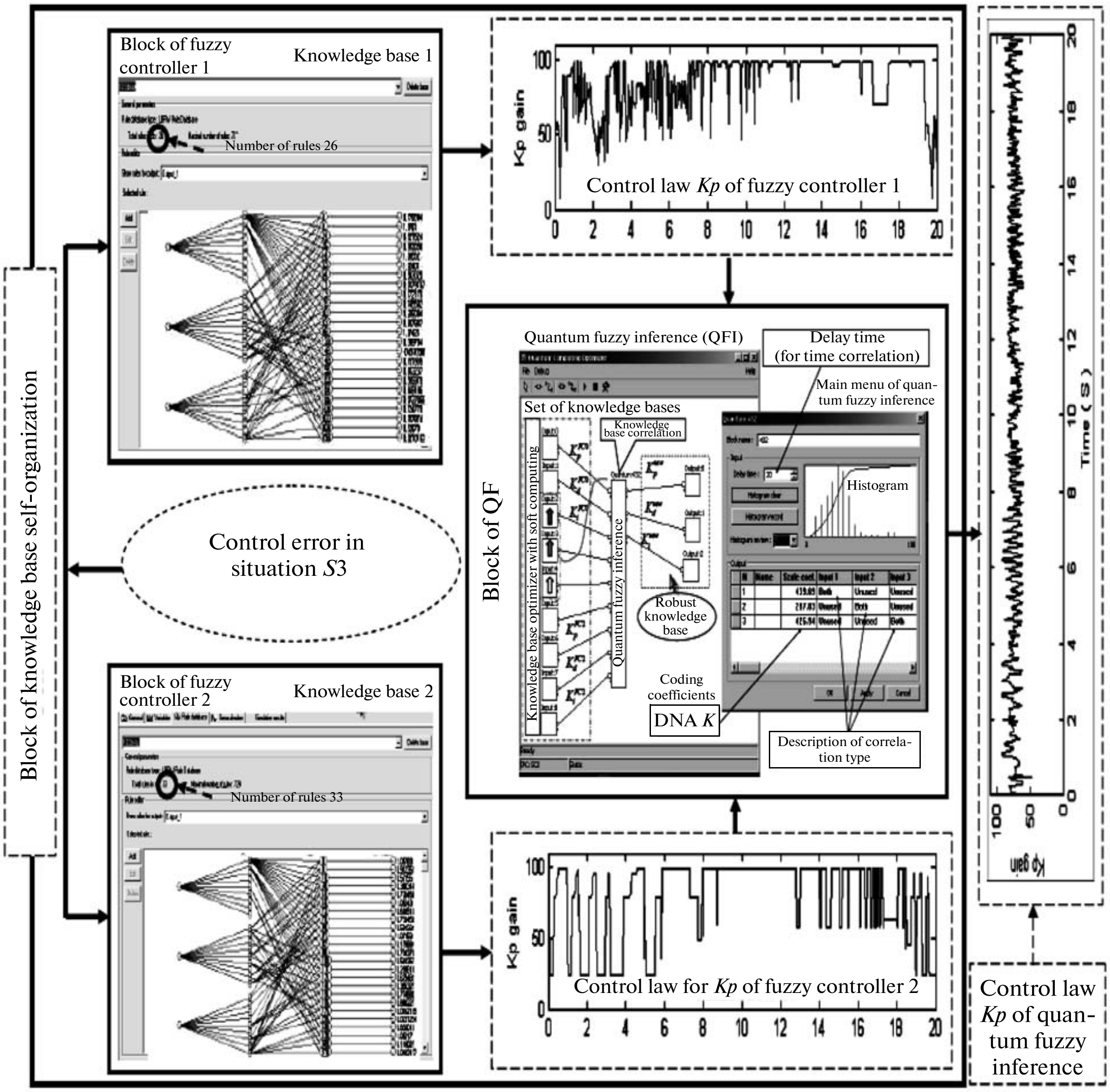


Fig. 18. Example of operation of the block of self-organization of the knowledge base based on quantum fuzzy inference.

where θ is the pendulum deviation angle (degrees); z is the movement of the cart (m); g is the acceleration of gravity (9.8 m/s^2); m_c is the pendulum mass (kg); l is the pendulum half-length (m); $\xi(t)$ is the stochastic impact; and u is the control force acting on the cart (N).

The equations for the entropy production rate in the control object and the PID controller have the following form, respectively [11]:

$$\frac{d}{dt}S_\theta = \frac{k\dot{\theta}^2 + \dfrac{ml\dot{\theta}^3\sin 2\theta}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2\theta}{m_c + m}\right)};$$
$$\frac{d}{dt}S_z = a_1\dot{z}^2; \quad \frac{d}{dt}S_u = k_d\dot{e}^2 \tag{4.2}$$

Figure 12a shows the law of summing these quantities (4.2) used in the structural diagram in Fig. 2. The following parameter values are determined: $m_c = 1$; $m = 0.1$; $l = 0.5$; $k = 0.4$; $a_1 = 0.1$; $a_2 = 5$; and the initial position $[\theta_0; \dot{\theta}_0; z_0; \dot{z}_0] = [10; 0.1; 0; 0]$ (the value of the pendulum deviation angle is given in degrees); the constraint on the control force is $-0.5 < u < 5.0$.

The specific feature of control problem for the given control object (4.1) is the application of one fuzzy PID controller for controlling the movement of the cart (with one degree of freedom), while the control object has two degrees of freedom.

The control goal is that the pendulum deviation angle (second generalized coordinate) reaches the

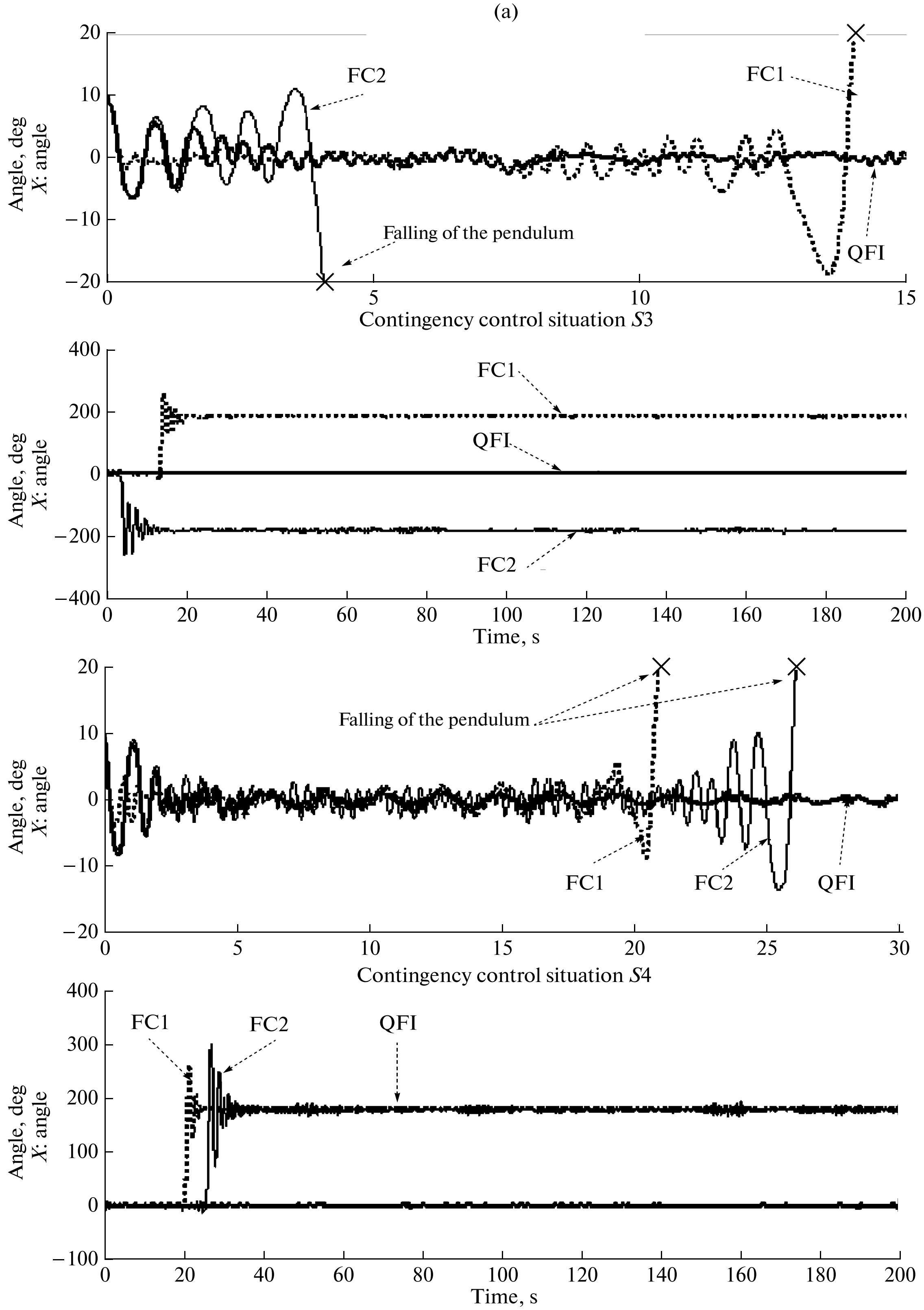
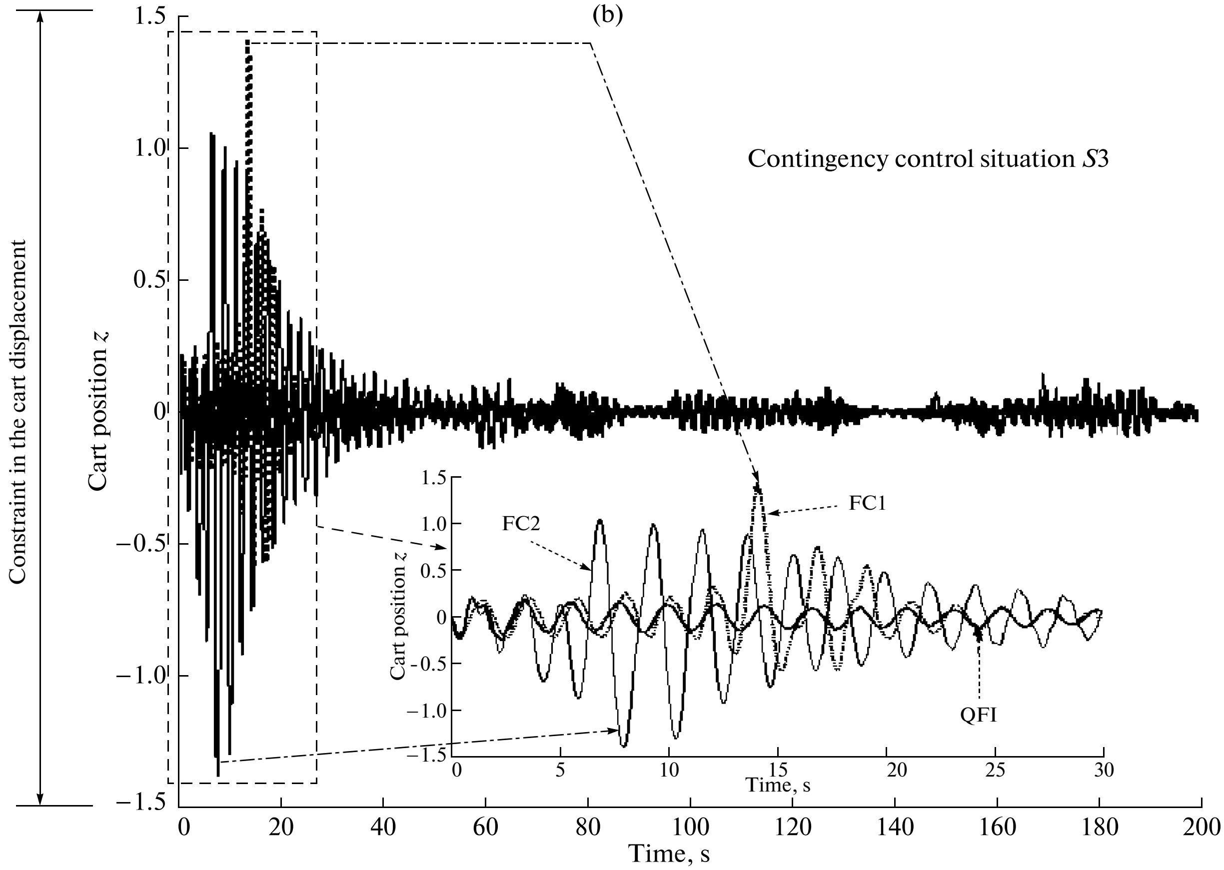


Fig. 19. Dynamic behavior of the control object and control laws of self-organized quantum controller, fuzzy controllers 1 and 2: (a) pendulum deviation angle in situations (**S3**, **S4**); (b) cart displacement in situations (**S3**, **S4**); (c) control laws of gains in situations (**S3**, **S4**).


(b)
Contingency control situation S3
Constraint in the cart displacement
Cart position z
FC1
FC2
QFI
Time, s
Time, s

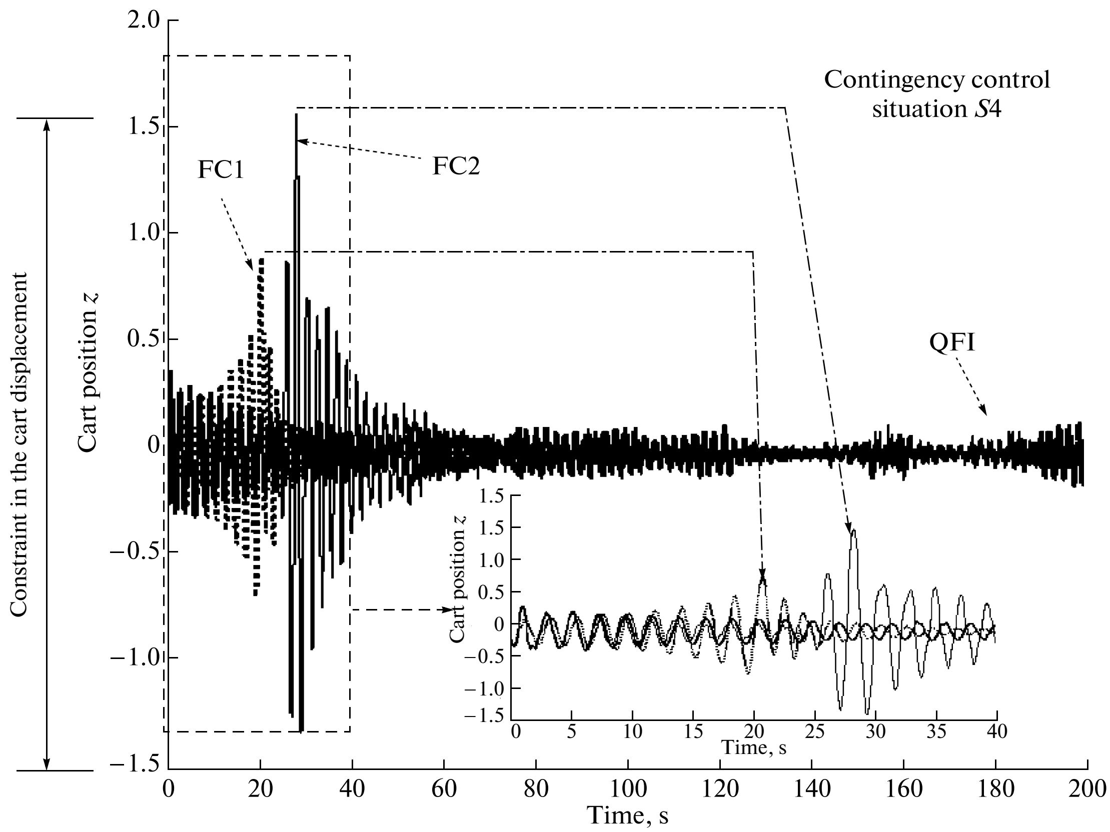

Contingency control situation S4
Constraint in the cart displacement
Cart position z
FC1
FC2
QFI
Time, s
Time, s

Fig. 19. (Contd.)

(c)

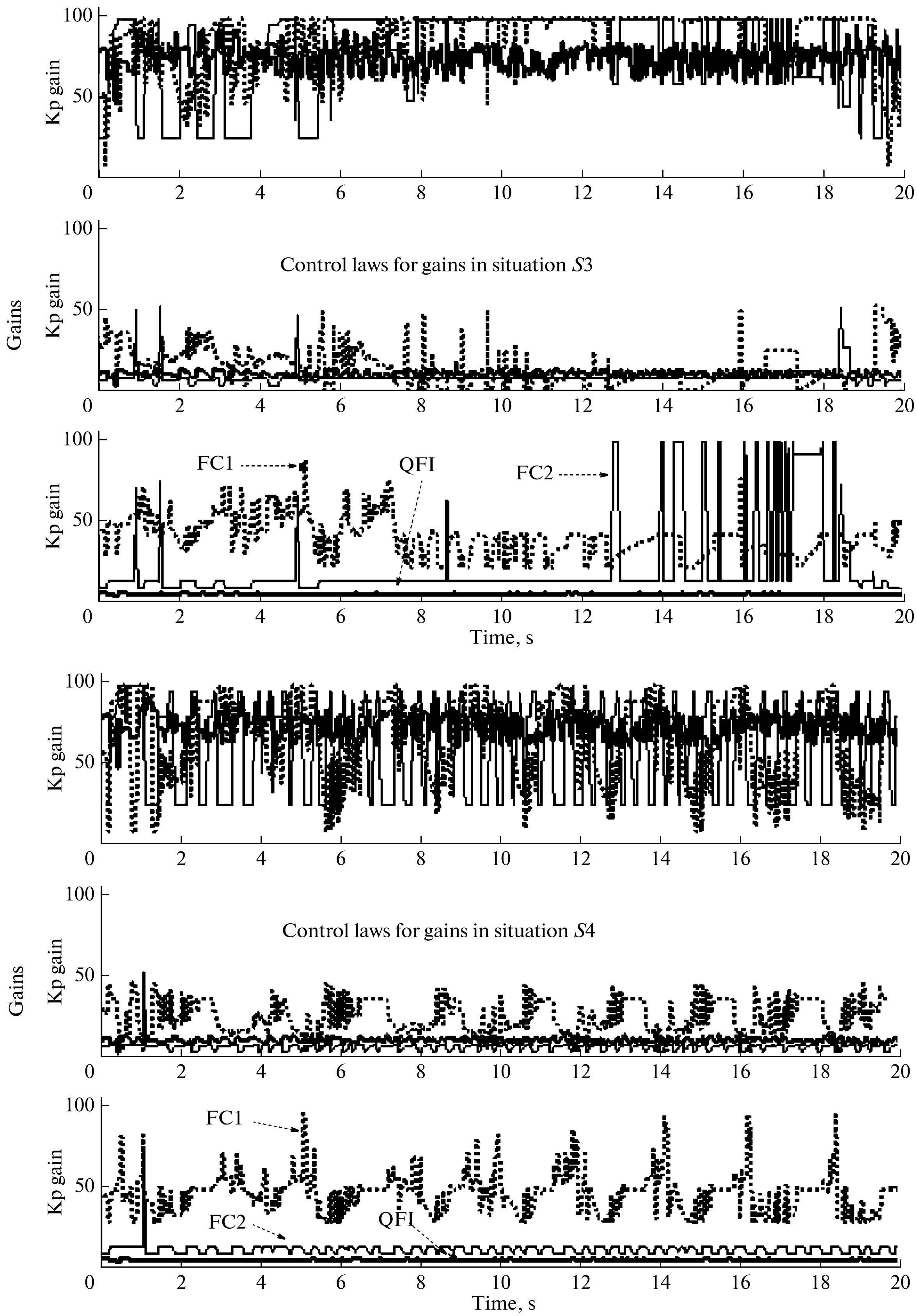


Fig. 19. (Contd.)

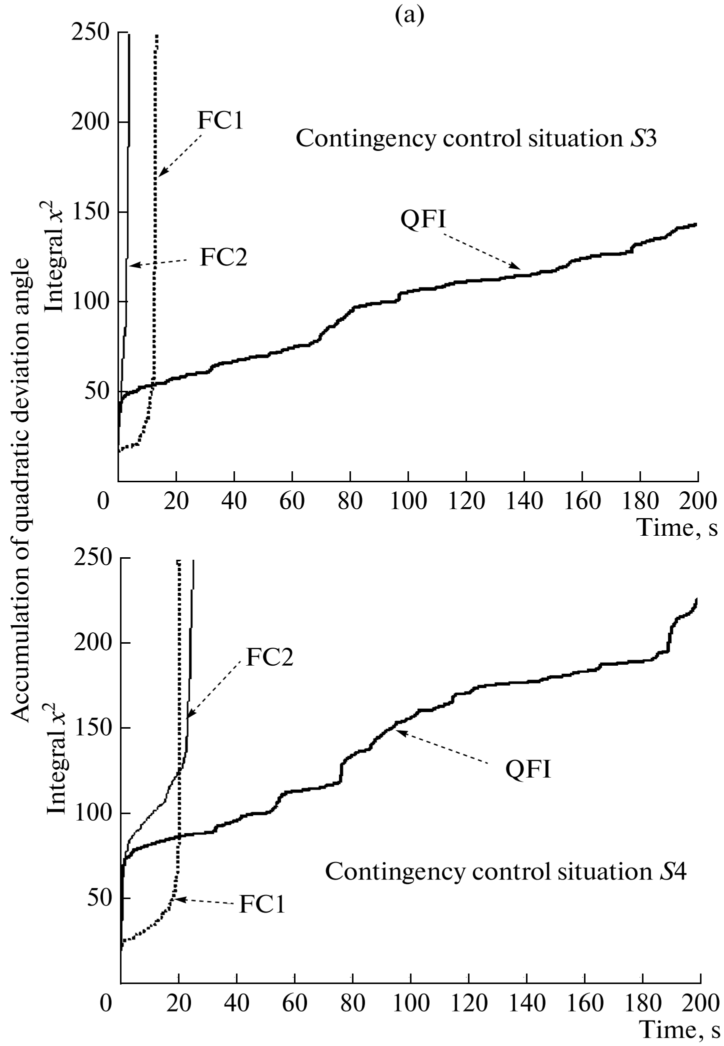


Fig. 20. Integral characteristics of dynamic behavior of the control object and control precision: (a) accumulation of quadratic deviation angle of the pendulum in situations (**S3**, **S4**); (b) accumulation of absolute value of the deviation angle of the pendulum in situations (**S3**, **S4)**; (c) accumulation of the value of quadratic control error in situations (**S3**, **S4**).

given value via the implicit control using the other generalized coordinate and corresponding essentially nonlinear cross-connections with the cart movement coordinate (effect of energy transmission [41] between the generalized coordinates).

In the case of the similar initial learning conditions, the knowledge base optimizer with soft computing is used to design knowledge base 1 of fuzzy controller 1 for the generalized criterion of minimal mean square error

$$\left[\int_{t_0}^{t_{end}} \theta^2 dt + \int_{t_0}^{t_{end}} \dot{\theta}^2(t) dt\right],$$

and knowledge base 2 for fuzzy controller 2 for the generalized criterion of minimal absolute error of the pendulum position

$$\left[\int_{t_0}^{t_{end}} |\theta(\tau)| d\tau + \int_{t_0}^{t_{end}} |\dot{\theta}(\tau)| d\tau\right].$$

The Gaussian noise was used as the random signal for designing knowledge base 1, and Rayleigh noise was used for forming knowledge base 2 (see Fig. 15, learning situations (**S1**, **S2**), respectively). Physically the first criterion is equivalent to the total energy of the overturned pendulum and the second criterion characterizes the precision of the dynamic behavior of the control object.

Figure 17 shows knowledge base 1 and knowledge base 2 with the corresponding activated numbers of rules equal to 22 and 33 for a total number of rules of 729.

Two contingency control situations (**S3**, **S4**) were simulated; in one of them (**S3**) the new noise $\xi(t)$ was introduced, the random signal with homogeneous one-dimensional distribution, the control error signal delay (0.03), and the noise signal in the position sensor of the pendulum (noise amplification coefficient 0.015). In situation **S4** the new noise was introduced: the random signal with the one-dimensional Gaussian

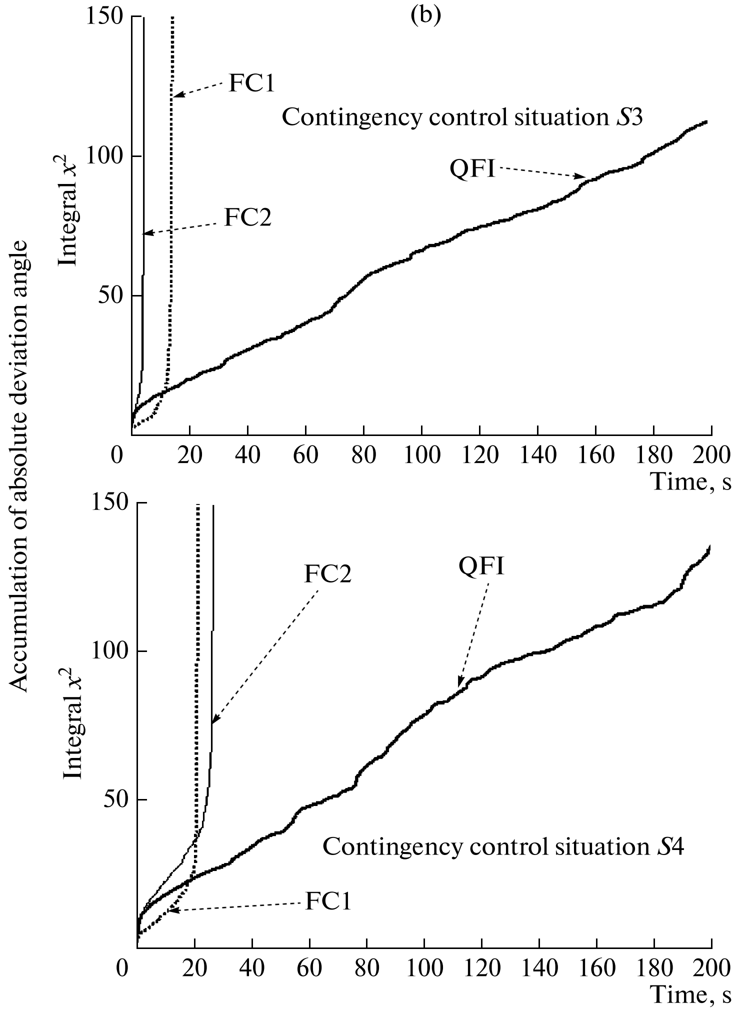


Fig. 20. (Contd.)

distribution, the delay of the control signal error (0.005 s) and the noise signal in the position sensor of the pendulum (noise amplification coefficient 0.01).

Figure 18 shows the example of operation of the quantum fuzzy controller for formation of the robust control signal using the proportional gain k_P in contingency control situation **S3**.

In this case, the output signals of knowledge bases 1 and 2 in the form of the response on the new control error in situation **S3** are received in the block of the quantum fuzzy controller. The output of the block of quantum fuzzy controller is the new signal for real-time control of the factor k_P. Thus, the blocks of knowledge bases 1, 2, and quantum fuzzy controller in Fig. 18 form the block of self-organization of the knowledge base in the contingency control situation.

Figure 19 shows the dynamic behavior of the studied system "moving cart–reversed pendulum" and the control laws of the self-organized quantum controller (quantum fuzzy inference), fuzzy controllers 1 and 2.

Remark 13. The following notation is used in Fig. 19 and below: $x = \theta$ is the angle of pendulum deviation from the given position; z is the cart position; quantum fuzzy controller is the quantum fuzzy controller with the spatial correlation based on the quantum fuzzy controller.

For a number of contingency control situations (for example, in the case of delay of the control signal error (0.001 s) and the noise signal in the position sensor of the pendulum (noise gain 0.01) in contingency control situation **S3**) the control algorithms for the knowledge base optimizer with soft computing are quite good and preserve the pendulum in the given position [7]. Figure 19 also shows a more complex contingency control situation when other control laws do not act, but can be used to design the robust control law.

The results of simulation (Fig. 19) demonstrate that the dynamic control object in contingency control situations (**S3**, **S4**) for the control of fuzzy controller 1 (fuzzy controller 2) loses stability, and for the control of quantum fuzzy controller the control sys-

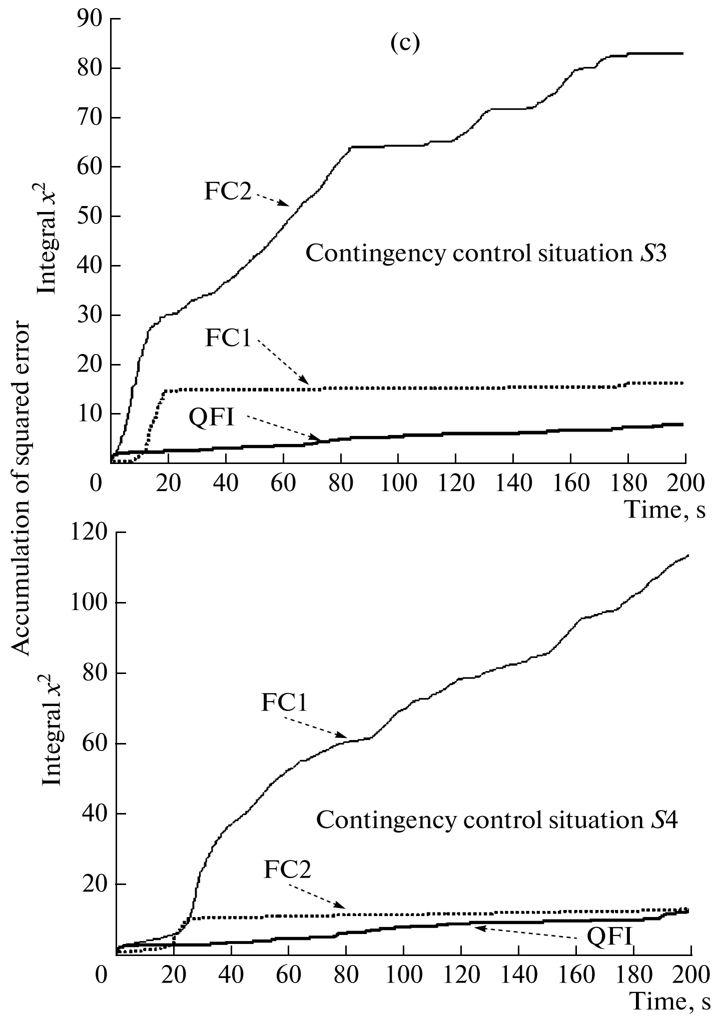


Fig. 20. (Contd.)

tem possesses the property of robustness and achieving the control goal is guaranteed.

Figure 20 shows the integral dynamic characteristics of the control object and the control precision.

According to the results of simulation (Fig. 20), the required amount of control for the given criteria in contingency control situations (**S3**, **S4**) for the control of fuzzy controller 1 and fuzzy controller 2 also is not achieved, while in the case of control of the quantum fuzzy controller the control system possesses the required amount of control.

This yields that two non-robust fuzzy controllers can be used to design in real time the robust fuzzy controller using quantum self-organization; the knowledge base of this robust fuzzy controller satisfies both quality criteria.

Therefore, the decomposition of the solution to the above multicriteria optimization problem for the robust knowledge base in the contingency control situation into partial solutions to optimization subproblems physically can be performed in real time in the form of separate responses of the corresponding individual knowledge bases optimized with different fixed cost functions and control situations. The aggregation of the obtained partial solutions in the form of the new robust knowledge base is performed based on the quantum fuzzy controller containing the mechanism of formation of the quantum correlation between the obtained partial solutions. As a result, only responses of the finite number of individual knowledge bases containing limiting admissible control laws in the given contingency situations are used.

The control laws of variation of the gains of the fuzzy PID controller formed by the new robust knowledge base have a simpler physical realization, and as a result they possess better characteristics of individual control cost function for the contingency control situation.

Thus, the output signal of the quantum fuzzy controller represents the real-time optimal control signal for variation of the gains of the fuzzy PID controller which includes the necessary (best) qualitative characteristics of output control signals of each of the fuzzy controllers with priority and dominating component among the control quality criteria, thus realizing the generalized self-organization principle.

This approach opens new prospects for application of the model of quantum fuzzy controller as the particular variant of the quantum self-organization algorithm (Fig. 2b) in multicriteria control problems for the control object with weakly formalized structure and large dimensionality of the phase space of control parameters, application of experimental data in the form of the learning signal without constructing the mathematical model of the control object. These facts present a great advantage which is manifested as the possibility of design of control with required robustness in real time.

CONCLUSIONS

The obtained results provide a more complete and clear understanding of the solution of the following complex problem which is of fundamental importance for the control theory and systems: the determination of the role and influence of analogues of quantum effects on the increased robustness of designed intelligent control processes. The model of the quantum control algorithm realizes the principle of (knowledge) self-organization according to the developed thermodynamic quality control criterion (minimal generalized entropy production). The technologies of intelligent computing (of the type of soft and quantum computing) make the basis for designing robust self-organized control systems under the conditions of contingency control situations. Quantum knowledge self-organization opens new prospects for application of the model of quantum fuzzy controller as the particular variant of the quantum algorithm of self-organization in multicriteria problems for control objects with weakly formalized structure and large dimensionality of the phase space of control parameters. In this case, it is possible to use experimental data in the form of the learning signal for construction of the mathematical model of the control object.

APPENDIX

Here, the notation and necessary results of linear algebra used in quantum computing, and the main operators of quantum algorithms are described.

Notation and rules of quantum computing. (1) By definition, if $Z = a + i \cdot b$, then Z^* is the complex conjugate, $Z^* = a - ib$.

(2) The vector $|\psi\rangle = \begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_n \end{bmatrix}$ is called the "cat vector", and $\langle\psi| = \left[c_1^*, c_2^*, \ldots, c_n^*\right]$, the "bra vector".

(3) $|\varphi\rangle$ is the internal product of the vectors $|\varphi\rangle$ and $|\psi\rangle - \langle\varphi|\psi\rangle$. In this case in quantum computing, this operation is defined in the complex space $\mathbb{C}^n$, rather than the space $\mathbb{R}^n$ of real numbers.

For example, if $|\varphi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix}$, $|\psi\rangle = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, then $\langle\varphi|\psi\rangle = [2, -6i]\begin{bmatrix} 3 \\ 5 \end{bmatrix} = 6 - 30i$.

(4) Two vectors are orthogonal if and only if $|\varphi\rangle \neq 0$, $|\psi\rangle \neq 0$, and $\langle\varphi|\psi\rangle = 0$. For example, the vectors $|\varphi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|\psi\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are orthogonal, since $[1^*, 0]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$. Similarly, the vectors $|\varphi\rangle = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $|\psi\rangle = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are orthogonal, i.e., the following condition is satisfied: $[1^*, 1^*]\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \times 1 + 1 \times (-1) = 0.$

(5) $|\varphi\rangle \otimes |\psi\rangle$ determines the tensor product and is often written as $|\varphi\rangle|\psi\rangle$. For example,

$$|\varphi\rangle|\psi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 3 \\ 2 \times 5 \\ 6i \times 3 \\ 6i \times 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 18i \\ 30i \end{bmatrix}.$$

Tensor product is the generalization of the bilinear operation of matrix multiplication

$$A\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} A \times a & A \times b \\ A \times c & A \times d \end{pmatrix};$$

it possesses the following properties:

$$(a)\ Rank(A \otimes B) = Rank(A) + Rank(B);$$

$$(b)\ Dimension(A \otimes B) = Dimension(A) \times Dimension(B).$$

Thus, tensor product can be used to exponentially extend the working space of calculations and form the *basis for parallel calculations.*

(6) A^* is the complex conjugate to A matrix. For example, if

$$A = \begin{bmatrix} 1 & 6i \\ 3i & 2 + 4i \end{bmatrix}, \text{ then } A^* = \begin{bmatrix} 1 & -6i \\ -3i & 2 - 4i \end{bmatrix}.$$

(7) A^{T} is the transposed to A matrix. For example, if

$$A = \begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}, \text{ then } A^{\mathrm{T}} = \begin{bmatrix} 1 & 3i \\ 6i & 2+4i \end{bmatrix}.$$

(8). $A^{\dagger}$ is the Hermitian conjugate to A matrix. Note that $A^{\dagger} = \left(A^{\mathrm{T}}\right)^*$. For example, if

$$A = \begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}, \text{ then } A^{\dagger} = \begin{bmatrix} 1 & -3i \\ -6i & 2-4i \end{bmatrix}.$$

(9) $\||\psi\rangle\|$ is the norm of the vector $|\psi\rangle$: $\||\psi\rangle\| = \sqrt{\langle\psi|\psi\rangle}$; it is used for normalization of $|\psi\rangle$.

(10) $\langle\varphi|A|\psi\rangle$ is the internal product of $|\varphi\rangle$ and $A|\psi\rangle$ or the internal product of $A^*|\varphi\rangle$ and $|\psi\rangle$.

(11) $|\varphi\rangle\langle\psi|$ is the external product of $|\varphi\rangle$ and $\langle\psi|$. For example,

$$|0\rangle\langle 1| = \begin{bmatrix} 1 \\ 0 \end{bmatrix}[0^* 1^*] = \begin{bmatrix} 1\times 0^* & 1\times 1^* \\ 0\times 0^* & 0\times 1^* \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and it is determined as the linear operator A which acts as follows: $(|\varphi\rangle\langle\psi|)(|w\rangle) = |\varphi\rangle\langle\psi|w\rangle = \langle\psi|w\rangle|\varphi\rangle$. Let us assume that $|w\rangle = \alpha|0\rangle + \beta|1\rangle$. We have $|1\rangle\langle 1|w\rangle = |1\rangle\langle 1|(\alpha|0\rangle + \beta|1\rangle) = \beta|1\rangle$.

(12) The projector P in the subspace $V_s = \{|00\rangle, |01\rangle\}$ of the vector space $V = \{|00\rangle, |01\rangle, |10\rangle, |11\rangle,\}$ is defined as

$$P\left(\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle\right) = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle.$$

The operator P projects the vector from V into the subspace V_s. The projector P is represented in the form of the external product. In the given subspace constructed on the orthonormal vectors $\{|u_1\rangle, |u_2\rangle, \ldots, |u_n\rangle\}$, the projector is written in the form of the sum of external products

$$P = \sum_{i=1}^{n} |u_i\rangle\langle u_i|.$$

Each component $|u\rangle\langle u|$ is Hermitian, and the operator P is also Hermitian, $P^{\dagger} = P, P^2 = P$, , and $Q = I - P$ is called the orthogonal supplement. If the projector $M_m = P$ is used for measurement, the probability of the result of measurement m is determined as $\mathrm{pr}(m) = \left\langle\psi\left|M_m^{\dagger}M\right|\psi\right\rangle = \langle\psi|M|\psi\rangle$.

(13) The commutator and anticommutator which establish the relation between the two operators A and B are written, respectively, as

$$[A, B] = AB - BA \text{ and } \{A, B\} = AB + BA.$$

The operators A and B commute (anticommute) if $[A, B] = 0$ ($\{A, B\} = 0$).

For understanding the operation of the main operators of the quantum algorithm we present the following examples.

E x a m p l e 3. *Quantum bit as a quantum state.* The classical bit can be in one of the two states, 0 or 1. Thus, its physical state can be represented as $b = a_1 0 + a_2 1$; it has one of the forms: either $a_1 = 1$ and $a_2 = 0$, then $b = 0$, or $a_1 = 0$ and $a_2 = 1$, then $b = 1$. The state of the quantum bit $|\psi\rangle$ is determined by the vector in the two-dimensional complex vector space. Here, the vector has two components and its projections onto the bases of the vector space are the complex numbers. The quantum bit ψ is represented (following Dirac, in the form of the *cat* vector) as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ or using the vector notation as $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \langle\psi| = [\alpha\,\beta]^T$ (*bra* vector). If $|\psi\rangle = |0\rangle$, then $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The amplitudes α and β are the complex numbers for which the following condition is satisfied: $\alpha\alpha^* + \beta\beta^* = 1$, where "*" is the operator of complex conjugation, $\{|0\rangle, |1\rangle\}$ for the pair of orthonormal basis vectors called the *states of the calculational basis*. If α or β take zero values, then ψ determines the classical pure state. In the opposite case, it is said that ψ is in the state of superposition of two *classical* basis states. Geometrically the quantum bit is in the continuous state between $|0\rangle$ and $|1\rangle$ until its state is measured. The idea of the probability amplitude of the quantum state is the combination of the concept of the state and phase. If the system consists of two quantum bits, then it is described as the tensor product. For example, in Dirac's notation the two-quantum bit system is determined as

$$|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1\psi_2\rangle = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle.$$

The number of possible states of the combined system exponentially increases if the quantum bit is added.

This results in the problem of estimation of quantum correlation which is present between the quantum bits in the compound system.

E x a m p l e 4. *Formation of the state of superposition using Hadamard (Walsh–Hadamard) operator.* The existence of the state of superposition and the effect of measurement of the quantum state physically means that there exists the *information hidden from the observer*, which is contained in the closed quantum system (until the time instant of its excitation from the external perturbation) in the form of observation of the quantum state.

The system remains closed while it interacts with the environment (i.e., while the system is observed). In this case, very important is the following question: *how can the information hidden in the superposition be effi-*

ciently used? In the conventional formalism of quantum computing quantum operators are described in the equivalent matrix form. The multiplication of the operator matrix by the state vector means the action of the operation on the studied system. For example, the action of the Hadamard matrix (H) on the system $|\psi\rangle = |0\rangle$ can be represented as

$$H|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Similarly,

$$H|\psi\rangle = H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\ 0\end{bmatrix} + \begin{bmatrix}0\\ -1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),$$

i.e., the Hadamard transformation generates the state of the quantum bit in the form of the superposition of two classical states. The formation of the superposition with equivalent probability amplitudes is the important step for many quantum algorithms. Applying $H^{\otimes n}$ for the corresponding basis states $|x\rangle \in \mathcal{H}_n$, $x \in \{0,1\}^n$, we obtain as a result the equivalent form of the Hadamard transformation

$$H|x\rangle = \frac{1}{\sqrt{2}}\sum_{z=0,1}(-1)^{x\cdot z}|z\rangle,$$

where $x \cdot z = x_1z_1 + \ldots + x_nz_n$ for $x = 0$ and $x = 1$.

Thus, the state of superposition with equivalent probability amplitudes $\frac{1}{\sqrt{2^n}}$ for each basis state is obtained by applying the operator $H^{\otimes n}$ to the state $|0\rangle$. The value of the state of superposition for the theory of computing processes becomes clearer if the resulting state of superposition is interpreted as the set of 2^n classical trajectories (paths) of calculations with equivalent weights for which the quantum computer executes parallel calculations. In this sense the state of superposition serves as the first step of organization of quantum parallelism.

E x a m p l e 5. *Quantum massive parallelism and models of computing with quantum oracle.* The term *"quantum massive parallelism"* is used for the description of the potential power of quantum computing based on the realization of parallel calculations, applying the architecture of one quantum computer. On the intuitive level the realization of parallel calculations seems possible using the network of parallel computers. Moreover, in classical computing the higher computer efficiency (in the form of the rate of exponential calculation time) requires exponential growth of the number of processors or the dimensionality of the physical space. However, since q-bit can represent the superposition of two different states, in quantum computing the linear growth of the dimensionality of the physical state results in the exponential effect of parallel calculations, and consequently, the exponential efficiency of the computing time. This effect is known as the "*quantum massive parallelism*".

In classical physics, two-dimensional vector objects form the $2n$-dimensional vector space. The q-bit is the basis in quantum computing and is represented in the form of the two-dimensional vector. Therefore, unlike the principles of classical computing, due to the quantum parallelism n q-bits form the 2^n-dimensional vector space due to the application of the tensor product. Such exponential extension of the possible physical state with respect to the classical computer results in the exponential growth of the rate of information processes of data processing in the quantum computer. Let us illustrate the concept of application of quantum massive parallelism to parallel calculations using the example of two-q-bit system. The set of initial q-bits in the superposition of the basis states contains all possible coded signals. Therefore, the single action of the transformation T can be used for generation of the set of output q-bits in the superposition of the basis states which represent all possible output signals. Therefore, we have

$$\underbrace{\begin{pmatrix}\delta_0\\ \delta_1\\ \delta_2\\ \delta_3\end{pmatrix}}_{\text{All inputs}} \quad \otimes \quad \underbrace{T}_{\text{Single transformation}} \quad = \quad \underbrace{\begin{pmatrix}\delta_0 \otimes T\\ \delta_1 \otimes T\\ \delta_2 \otimes T\\ \delta_3 \otimes T\end{pmatrix}}_{\text{All outputs}}.$$

Therefore, all output states can be calculated simultaneously, and the application of the measurement probability in the quantum computer yields the result of parallel calculation in the form of the single signal similar to the calculation using the classical computer. All outputs are obtained using just *one* transformation, but only one output result from the superposition of possible results is extracted in the course of measurement.

The particular extracted result has the probabilistic character. Thus, the quantum algorithm should use the quantum massive parallelism in such a way that in the course of measurement the designed result is extracted. This is a difficult task, which cannot be solved using classical programming.

Two approaches to the solution of this problem are known: (1) all mutual properties of output signals are measured; it is used in Schor's factorization algorithm for measurement of the period of output signals; (2) the technique of increasing amplitudes of basis states for extracting the sought solutions with high probability; it is used in Grover's quantum search algorithm.

Example 6. *Notion and definition of "intelligent" quantum state.* Let us consider the possible approach to the choice of the criterion of optimization of extracting the priority state from the formed superposition of coded possible states. For this purpose, the notion of the "intelligent quantum state" is used which was introduced in the quantum measurement theory as the states with minimal uncertainty (in the sense of minimum of Heisenberg's uncertainty inequality). This notion is also connected with the solution of quantum wave equations (of the type of Schrödinger equation, etc.) for which the wave packet of the state of the quantum system is the coherent state. For this state, the uncertainty relation achieves the global minimum. The definition and calculation of the state in the quantum algorithm is given based on the definition of von Neumann entropy and Shannon information entropy in this quantum state. In this case the "intelligent quantum state" in the quantum algorithm is the minimum of the difference between the information entropy of the Shannon quantum state and the physical entropy of the Neumann quantum state,

$$\mathbb{I}\left(\left|\text{Quantum state}\right\rangle\right) = \min\left(H^{Sh} - S^{vN}\right), \quad (A.1)$$

where H^{Sh} and S^{vN} are the Shannon and Neumann entropies, respectively. According to the laws of quantum information theory, we obtain the following inequality:

$$H^{Sh} \geq S^{vN},$$

$$\text{i.e. } \mathbb{I}\left(\left|Quantum\ \ state\right\rangle\right) \geq 0. \quad (A.2)$$

We recall that the squared probability amplitude of the state in quantum mechanics is equal to the classical probability of finding the quantum system in the particular state (Bohr's postulate which has several variants of strict substantiation). From the point of view of the quantum information theory the pure quantum state is known to be characterized by the zero value of the von Neumann entropy. Therefore, the "intelligent" quantum state in the considered quantum algorithm is determined via the calculation of the minimum of the Shannon information entropy. The sought minimum is achieved for the maximal state probability (by definition of the Shannon information entropy of the quantum state,

$$H^{Sh} = -\sum_i p_i \ln p_i,$$

i.e., the global minimum is observed for the maximal probability p_i). Since p_i by definition is the squared probability amplitude, the maximum principle of the probability amplitude in the case of the correlated state can be taken as the criterion of selection of the priority "intelligent" correlation (coherent) state in the superposition of possible candidates.

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