Soft computing simulation design of intelligent control systems in micro-nano-robotics and mechatronics


Abstract The soft computing simulation design methodology of intelligent control system for mobile micro-nano-robots based on modeling of non-linear dissipative equations of robots motion with a minimum entropy production is described. It includes hierarchical levels for description of dynamic behavior of mobile micro-nano-robots based on laws of microphysics, quantum logic of intelligent dynamic behavior of control objects, optimal control of states and dynamic system theory of mechanical motion. The description of a thermodynamic intelligent behavior (with minimum entropy production) of control objects (robots) and their interrelations with Lyapunov stability conditions are introduced. The role of soft computing on the basis of GA with a fitness function as a minimum entropy production for intelligent control of mobile micro-nano-robots is discussed.

Key words Micro-nano robots, Soft computing, Entropy production rate, Lyapunov stability

1 Introduction

Micromechatronics is the synergetic integration of both mechanical and electronic systems based on scaling effects in the micro world [6]. The minimum feature size of these structures is on the order of μm and the maximum total size is on the order of mm [13]. When separations between objects are small enough, certain thermodynamics and quantum effects become manifestly significant even if the masses of the objects are too large by quantum standards [11]. When the control objects are minituarized, physical phenomenon is different from macroworld. Viscosity and friction forces become more effective than an inertial force and it is needed to take into account thermodynamic effects from dissipative processes. As the dimensions between components of micro-mechanical systems decrease, the need for better understanding of interactions between micro-surfaces at small separations is appeared. The Casimir effect, for example, is the attractive pressure between two flat parallel plates of solids that arises from quantum fluctuations in the ground state of the electromagnetic field. The magnitude of this pressure varies as the inverse fourth power of the separation between the plates. This force has all quantum mechanical properties in origin and may be attractive or repulsive, depending on the geometry of the surfaces. At the 20 nm separation between two metallic plates, the attraction is approximately 0.08 atmosphere [16]. For this case the parallel plate Casimir pressure is identical to the retarded van der Waals attractive pressure between two parallel plates. These effects are very important for describing of a micro manipulation based on the microphysics. In these cases scaling effects require different approaches to micro mechatronics and conventional mechatronics [6, 8].

In this paper we describe some particularities of modeling methodology in R & D of mobile micro-nano-robots. Simulation results of intelligent thermodynamic behavior (with minimum entropy production in a control object and a control system) of micro robot in a fluid [5, 17] are introduced.

2 Modeling of mobile micro-robots as thermodynamic open systems

Consider some peculiarities and typical examples of mobile micro-robots modeling in accordance with the developed methodology.

Entropy production and dynamic stability of a micro-robot in a fluid

Consider a mobile micro-robot (MMR) (prototype of that was developed in [5] and has a new steering mechanism) as an open thermodynamic macro physical system. The
mathematical model of the micro-robot as a mechanical system in a fluid is described as following [5]:

\[
m_1\ddot{x}_1 + C_d \frac{\rho}{2} A_1 |\dot{x}_1| \dot{x}_1 + K_1 (x_1 - x_0 - l_1 \theta_0) \\
- K_2 (x_2 - x_1 - l_2 \theta_1) = \tau(t) + \zeta(t),
\]

\[
m_2\ddot{x}_2 + C_d \frac{\rho}{2} A_2 |\dot{x}_2| \dot{x}_2 + K_2 (x_2 - x_1 - l_2 \theta_1) = 0
\]

\[
m_3\ddot{x}_3 + C_d \frac{\rho}{2} A_3 |\dot{x}_3| \dot{x}_3 + K_3 (x_3 - x_2 - l_3 \theta_2) = 0
\]

where \( \theta_{n+1} = -\frac{1}{\tau_n} \frac{1}{\tau_n} (x_{n+1} - x_n) \) and \( m_1 = 1.6 \times 10^{-7} \) kg; \( m_2 = 1.4 \times 10^{-6} \) kg; \( m_3 = 2.4 \times 10^{-6} \) kg; \( l_1 = 2.0 \times 10^{-3} \) m; \( l_2 = 4.0 \times 10^{-3} \) m; \( l_3 = 4.0 \times 10^{-3} \) m; \( K_1 = 61.1 \) N/m; \( K_2 = 13.7 \) N/m; \( K_3 = 23.5 \) N/m; \( A_1 = 4.0 \times 10^{-6} \) m²; \( A_2 = 2.4 \times 10^{-5} \) m²; \( A_3 = 4.0 \times 10^{-5} \) m²; \( C_d = 1.12 \); \( \rho = 1000 \) kg/m³; \( \zeta(t) \) - stochastic excitation (white noise).

The thermodynamic equation for the entropy production of the MMR is defined from Eq. (1) as

\[
\frac{dS}{dt} = \sum_{i=1}^{3} m_i \frac{\rho}{2} A_n |\dot{x}_n| \dot{x}_n^2, \quad i = 1, 2, 3,
\]

and the Lyapunov function is described as follows [17]

\[
V = \sum_{i=1}^{3} \frac{m_i \dot{x}_i^2}{2} + \sum_{i=1}^{3} \frac{K_i (x_i - x_{i-1} - l_i \theta_{i-1})^2}{2} + \frac{S^2}{2},
\]

where \( S = S_i - S_c \) and \( S_c \) is the entropy of controller with torque \( \tau \) in Eq. (1).

For the stability analysis and computer simulation of a dynamic behavior of the MMR Eq. (1) is written in traditional form of differential equations as \( \ddot{x}_i = \varphi(x_i, \tau, t) \). Here we use the following relation between Lyapunov function and the entropy production for the micro-robot as an open system [17]

\[
\frac{dV}{dt} = \sum_i \varphi_i(x_i, \tau, t) x_i + (S_i - S_c) \left( \frac{dS_i}{dt} - \frac{dS_c}{dt} \right) < 0
\]

From Eq. (4) the necessary and sufficient conditions for Lyapunov stability of the MMR is expressed as follows:

\[
\sum_i \varphi_i(x_i, \tau, t) x_i < (S_i - S_c) \left( \frac{dS_i}{dt} - \frac{dS_c}{dt} \right), \quad \frac{dS_i}{dt} > \frac{dS_c}{dt},
\]

i.e., the stability of a MMR’s motion can be achieved with “negentropy” (by Brillouin’s terminology) \( S_c \) and the change of entropy \( dS_i/dt \) in motion of the MMR must be subtracted from the change of entropy \( dS_i/dt \) in a control system in accordance with Eq. (5).

Figure 1 shows the thermodynamic behavior of the MMR with intelligent and conventional PID-controller accordingly. For the simulation of a dynamic behavior the GA (genetic algorithm) with the fitness function as the minimum entropy production in the motion of the micro-robot and in the PID-controller was used. Figures 1 and 2 show that according to the conditions (5) for Lyapunov stability the intelligent behavior of the MMR with the principle of minimum entropy production can be achieved.

### 3 Mobile nano-robots in a quantum stochastic micro world

Mobile nano-robots (MNR’s) are widely used in medicine and biology [4]. MNR’s size and working conditions of artificial life [17] are compared in this case with a quantum molecular level [11]. A non-equilibrium stable dynamic behavior of a MNR may be realized through a self-organization mechanism between the MNR motion and a fluid (biological) medium. For the description of this self-organization mechanism in non-equilibrium systems (as MNR’s in a fluid medium) heat fluctuations and dissipative quantum processes must be taken into account [10, 11].

In this article we will consider the intermediate case – a control of a MMR’s motion in a quantum bio-molecular medium. In this case the cooperative system “a mobile micro-robot + a quantum bio-molecular medium” can be considered as a mobile nano-robot (MNR).

The use of MMR’s (at a classical corpuscular level) in bio-molecular medium (at a quantum wave macro-physic level) leads to the necessity of investigation of a correlation between classical and quantum levels of a cooperation system.

#### 3.1 Qualitative description of MNR dynamic problem

In this article we present in particular the models describing biological membranes and a MMR’s dynamics in a quantum fluid. The molecular membrane that bounds both a cytoplasm of a cell as a whole and individual organelles contained within it is a structure common to all living systems and biologically inspired evolutionary systems. All communication of a cell or an organelle with its environment is carried out across the membrane: that includes ion transport processes, diffusive transport of small molecules (such as H₂O and CO₂), as well as a transport of large molecules such as lipoprotein [2]. Many biologically inspired evolutionary systems and biological processes are associated with an energy transfer through protein, where this energy is realized by a hydrolysis of adenosine triphosphate (ATP) [3, 19].

We will discuss the possibility of a phonon mechanism creation as the analogy to Davydov soliton excitation [3, 19] on the basis of a control of a MMR’s vibration in a fluid biological medium (as an analogy of Eq. (1)) for a transfer without loss of energy and for a transport of the MMR itself in biologically inspired evolutionary systems.

We will introduce a qualitative analysis of the MMR’s dynamic behavior on the basis of physical representations (quantum mechanics and thermodynamics) by Schrödinger and von Neumann. The mathematical background of this qualitative analysis is mathematical models of quantum mechanics (soliton-like solutions) and non-equilibrium thermodynamics (self-organization structures.
Fig. 1a–h. Simulation results of dynamic and thermodynamic behavior of MMR under different control strategies. Curve 1 – GA-PID controller with minimum of entropy production, curve 2 – Conventional PID controller

Fig. 2a–c. Simulation results of dynamic and thermodynamic behavior of MMR under different control strategies in 3d space. Curve 1 – GA-PID – controller with minimum of entropy production, curve 2 – Conventional PID controller
through chaos and dissipative processes in open dynamic systems with entropy exchange [11, 17]). In the general case a new (Lie-admissible non-Hermitian isotopic) non-linear model of quantum mechanics with entropy exchange can be used [7, 17] (see Appendix for details). A characteristic feature of this investigation is the study of a cooperative self-organization mechanism for the creation of artificial life conditions of a biological MMR on the basis of a soft computing for the control of a correlation between a classical level (Newton mechanics) and a quantum macro-physical level (molecular fluid medium).

The mathematical model of a MMR’s interaction with a quantum fluid medium is developed. Quantum micro- and nano-robots and their interactions with stochastic environments of quantum systems are described. Equations of cooperative system motion with entropy exchange based on non-linear stochastic Schrödinger equations in [11] are in detail described. A quantum robot in general case is a mobile quantum system that includes an on-board quantum computer and ancillary systems [1]. The study of quantum computation based on the increased efficiency of quantum computation compared to classical computation for solving some important intelligent control problems in [18] are discussed. The simulation results of MNR’s motion are examined.

3.2 Biological, physical and mathematical models
We will discuss a thermodynamic interaction between a quantum macro-physical (molecular membranes) level and a classical level in mechanical motion of a cooperative system (a MMR and a fluid medium) [17].

3.2.1 Biological membrane models
A membrane is essentially a liquid bilayer with a variety of other molecules (e.g., globular proteins, cholesteryl) embedded therein and forming a complex interacting system. Such membrane macromolecules as α-helices and β-sheets are instrumental in transmembrane transport processes [2, 3, 19]. The most numerous membrane constituents, phospholipids, consist of a polar head group based on the phosphate and attached through a glycerol moiety to two hydrocarbon chains containing 14–20 carbon atoms [2]. Thus, in a water the phospholipid forms a bilayered lamina 40–50 Å thick, with the polar head group facing outwards and tails inwards, away from the water (Fig. 3 [2]).

The electric charge in the head group makes it hydrophobic, while the oily hydrocarbon chains are hydrophobic. The lamina close into stable, spherical vesicles to completely isolate the chains from the membrane’s hydrated environment (Fig. 4 [2]). Each of the two lipid layers of the membrane may be have an independent phospholipid composition. The membranes’ rigidity and configuration change with the temperature. At a narrow temperature rate, in particular, the membrane undergoes a sudden pronounced loss in packing density becoming more fluid-like above this characteristic temperature $T_{c}$, with individual lipids undergoing Brownian motion and exchanging with other on the order of a microsecond.

![Fig. 3. Schematic illustration of the membrane as a lipid bilayer](image)

![Fig. 4. Graphical representation of the spherical membrane](image)

The fluid-like quality of the membrane is enhanced by the presence of cholesterol molecules amidst hydrocarbon chains, since they reduce chain–chain attraction. Metabolically active biological cells should exhibit, according Fröhlich theory, long-range coherence manifested by Bose condensation of longitudinal elastic vibrations of membrane dipoles into a narrow (microwave) frequency band. The ensuing dynamical pictures involve in this case high-frequency longitudinal dipolar oscillations of membrane segments with displacements perpendicular to the surface. Their frequency can be estimated as $\omega = (3–8)10^{11}$ Hz. A potential difference of about 10–100 mV is known to exist across biological membranes resulting in an electric state in parts of the membrane. Dipole oscillations within large molecules such as DNA, RNA, proteins, and hydrogen-bonded amides are capable in this range. The lipid head groups of the cell membrane are less massive and appear to have fewer degrees of freedom than the tail groups. They are also more regular in their structure.

3.2.2 Physical model
It is expected that the system comprised of the membrane itself, the energy pumps, and the environment can be adequately described by an appropriate Hamiltonian. The membrane will be modeled as a collection of phospholipids. The hydrocarbon tails can be considered separately from the lipid heads, which is based on the hierarchy of relaxation times. In this case the tails predominantly determine
the equilibrium phase of the membrane while the heads are crucial to the dynamics of the system. Lipid head groups exert lateral pressure on the tails, thereby affecting the equilibrium properties of the membranes as whole. In dynamical picture the tail appears, however, as nearly “frozen” and they determine the equilibrium spacings between the head groups as a function of a temperature. A model Hamiltonian is proposed in [2] that involves the oscillations of both the lipid head groups and hydrocarbon chains of membrane. Opposite limiting case (to Frölich theory) assume the long-wavelength approximation and neglect the dispersion in the non-linear part of Hamiltonian (Davydov region). Following Davydov’s idea [3] one can take into account the coupling between the amide-I vibration and the acoustic phonons on the lattice. Through this coupling non-linear terms appear in the equations of motion. In this way the energy can be transported in solitonic wave. Davydov [3] has shown that the localized collective state on the solitary wave type (soliton) exists in quasi one-dimensional (1D) molecular chains by balancing the effects of non-linearity against those of dispersion. The Davydov soliton travels as a solitary wave retaining its form and energy without dissipation. It usually exhibits remarkable stability and particle properties even after collision.

In this paper we will discuss the possibility of preparing phonon mechanism analogy of soliton excitation on basis of a MMR’s vibration in a quantum fluid medium. Firstly, let us discuss a main idea of Davydov model of soliton.

In the quasicontinuous approximation we introduce the displacement $\beta(x, t)$ of molecules from equilibrium in the language of coherent states and the probability amplitude $\Psi(x, t)$ of a soliton excitation. Then the following [17, 19] below equation can be obtained:

$$\left[ i\hbar \frac{\partial}{\partial t} - \Lambda + a^2 J \frac{\partial^2}{\partial x^2} - 2 \lambda a \frac{\partial}{\partial x} \beta(x, t) \right] \psi(x, t) = 0 ,$$

$$\left[ \frac{\partial^2}{\partial t^2} - v_0 \frac{\partial^2}{\partial x^2} \right] \beta(x, t) - 2 \lambda a \frac{\partial}{\partial x} |\Psi(x, t)|^2 = 0 ,$$

where $\Lambda = \Xi + \omega - 2J$, $\Xi = E - D$, and $E$ is the excitation energy of an isolated molecule, $D$ is the deformation energy, $J$ is the transfer integral, $M$ is the molecular mass, $a$ is a lattice constant, $v_0^2 = a^2 \omega^2 / M$, $\omega$ is the longitudinal elasticity of the chain, $\chi$ is the coupling constant between the intermolecular excitation and molecular displacement. Under suitable initial conditions Davydov gave [3] a traveling solution of Eqs. (6), (7) as

$$|\Psi(x, t)|^2 = \frac{1}{2} \sinh^2 [\mu(x - x_0 - vt)] ;$$

$$\mu = \chi^2 [a \omega J] (1 - s^2) ;$$

$$\beta(x, t) = - \frac{\chi}{\omega} \tanh [\mu(x - x_0 - vt)] ;$$

$$s^2 = \frac{v_0^2}{v_0^2} .$$

Remark 1 Equation (6) describes a soliton motion (8) and has an external force as a non-linear parametrical excitation $\beta(x, t)$ (displacement of molecules from equilibrium).

Below, we will use this idea for preparing a phonon mechanism analogy (7) of a soliton excitation on the basis of a biological MNR vibration, and we will consider two cases: a without loss energy transfer and a transport of a MNR itself in a soliton body in a biologically inspired evolution system.

### 3.2.3 Mathematical model of a mobile micro-robot’s interaction with a fluid quantum medium

Consider one prototype of a MMR which is the MMR in a water (double-fin fish robot) that possesses a pair of fins and moves them symmetrically [5]. Therefore, the momentum of this robot is canceled and the tendency to move straight ahead is improved. Equations of motion of a cooperative system with entropy exchange [17] are following:

- **MMR motion (on classical level):**

$$\ddot{\xi} + F(\xi, \dot{\xi}, t, S) + \frac{dV_0(\xi)}{d\xi} = u(t)$$

(10)

- **MMR coupled motion (on quantum level in Schrödinger picture) (non-linear Schrödinger-type equation with dissipative processes and entropy exchange, obtained from quantum postulant [11]):**

$$\frac{i}{\hbar} \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + \left\{ V_0(x) + F(\xi, \dot{\xi}, t, S) \right\} |x - \xi(t)|$$

$$+ \frac{\gamma \hbar}{i} \left( \langle \ln \Psi \rangle - \langle \ln \Psi^* \rangle \right) \Psi - 2\chi a \frac{\partial^2 \Psi}{\partial x^2} \Psi = H_0 \Psi$$

Classical motion

(11)

- **Fluid medium (on quantum level) (non-linear Schrödinger-type equation with dissipative processes)**

$$\frac{\partial \Psi}{\partial t} = (E + \Delta E) \chi - R_0 J(\omega) \frac{\partial^2 \chi}{\partial x^2} - G \chi^2 \chi - \frac{\partial}{\partial \chi} Z \chi$$

$$+ F_j - \frac{2\chi a}{M \partial x} |\Psi|^2$$

(12)

Parameters $E, \Delta E, R_0, J(\omega), G, Z, \chi$ and $\gamma$ in [17] are defined. If we use the simple phase-shift transformation $\chi = \exp \{- (Z/2)t\} \Psi_1(x, t)$ then obtain new simple form of equation:

$$i\hbar \frac{\partial \Psi_1}{\partial t} = (E + \Delta E) \Psi_1 - R_0 J(\omega) \frac{\partial^2 \Psi_1}{\partial x^2} - G |\Psi_1|^2 \Psi_1$$

$$+ F_j - \frac{2\chi a}{M \partial x} |\Psi|^2$$

(13)

- **Entropy exchange (between a quantum fluid medium and MMR):**

$$S = S_{pr} - S_{ex} ,$$

(14)

$$\frac{dS_{pr}}{dt} = \frac{1}{T} F(\xi, \dot{\xi}, t, S_{pr}) \dot{\xi} ,$$

(15)

$$\frac{dS_{ex}}{dt} = \Theta(\xi, \dot{\xi}, t, S_1) ;$$

$$S_1 = -|\Psi|^2 \ln |\Psi|^2 = -\text{tr}(\rho \ln \rho) ,$$

(16)

where $S_{pr}$ – the entropy production of the MMR motion; $S_{ex}$ – the entropy exchange between the MMR and fluid.
medium; $S$ – the entropy of fluid medium (von Neumann entropy); $T$ – the temperature (constant); $m, \gamma$ – real constants.

Remark 2 In Eq. (11) we have the wave quantum mechanics description with dissipative processes and entropy production from a MMR motion on the classical level (Newtonian mechanics and phenomenological thermodynamics description), and coupled with dissipative processes in a fluid medium on the quantum macrophysics level. Equation (10) for MMR motion includes the entropy $S$ as a scalar parameter. The entropy production $S_p$ in Eq. (15) describes dissipative processes in MMR, determined from phenomenological thermodynamics, and defined as a single valued function of parameters in Eq. (10).

The entropy production $S_p$ of MMR in a fluid medium described on the quantum level using the density matrix $\rho$ and wave function $\Psi(x, t)$ (von Neumann entropy) as the solution of Eq. (11). If the coefficient of dissipative processes $\gamma \equiv 0$ then from Eq. (11) we obtain the result of paper [12]. If dissipative processes $F(\cdot)$ in Eq. (11) are $F(\cdot) \equiv 0$ then we obtain the result of paper [15].

The system of Eqs. (10)–(16) describes cooperative correlations between classical (MMR) and quantum macrophysics (a fluid medium) levels with an entropy exchange in a non-equilibrium dynamic behavior of the interaction of the MMR with the fluid medium.

### 3.2.4 The analysis of mobile micro-robot and quantum fluid medium motion

The system of Eqs. (11), (12) is the system of coupled nonlinear Shrödinger equations with cubic non-linearities, and has a solution as local solitons. The vibration of the MMR in the fluid medium as the solution of Eqs. (10), (11) influences on a wave function $\Psi(x, \xi, t)$ (as the solution of Eq. (11)). In an initial state the MMR has a coherent state in accordance to the classical equation of motion as:

$$\Psi(x, t) = \langle x | \Psi(0) \rangle = \frac{1}{\sqrt{2\pi d^2}} \exp \left\{ i k_0 \xi - \frac{x^2}{4d} \right\}$$

and Eq. (12) has a solution as local soliton

$$\chi(x, t) = \frac{[N \mu \exp \{-Zt\} \exp[i(kx - \omega_0 t)]}{\cosh((\mu/R_0) \exp(-Zt)(x - x_0 - vt))},$$

where $\mu(x, t) = \mu_0 \exp(-Zt)$, $\mu_0 = \frac{k_0}{2|\mu|} N$, $N = \int_{-\infty}^{\infty} |\chi(x, t)|^2 \, dx$, $N$ – the occupation number of interacting quantum modes, $k$ – wave number.

Equation (11) is the non-linear dissipative Shrödinger-type equation and has in case of a soliton-like solution (as a result of a cooperative effect of correlation between classical and quantum levels). We found the solution as

$$\Psi = N \exp(R + iM), \quad \xi = x - \xi(t).$$

If we introduce the ansatz [15, 17]

$$M(t, x, \xi) = K(t) + L_x(t)\xi + M_x \xi^2 = M(t, x), \quad R(t, x, \xi) = - \left( P(\xi - \xi(t))^2 - \frac{\gamma}{2} \right) = R(t, x),$$

where

$$K(t) = -\frac{1}{\hbar} \int_0^t \Phi(\xi, \xi, \tau) \, d\tau - \frac{1}{4} \frac{\omega_0^2}{\Omega} t,$$

$$\Phi(\xi, \xi, t) = \frac{1}{2} m \xi^2(t) + \frac{1}{2} m \omega_0^2 \xi^2(t); \quad \frac{\hbar}{\omega_0} \xi + \gamma \xi + F(\xi, \xi, t) + \omega_0^2 \xi = 0,$$

$$L(t) = \frac{m}{\hbar} \xi(t), \quad M(t) = -\frac{m \gamma}{4\hbar},$$

$$P = \frac{m}{2\hbar} \Omega, \quad \Omega = \left( \frac{\omega_0^2}{2} - \frac{\gamma^2}{4} \right)^{1/2},$$

then the soliton-like solution has the following form

$$\Psi(x, t, \xi(t)) = \left( \frac{m\Omega}{\pi \hbar} \right)^{i} \times \left( \exp \left\{ \frac{i}{\hbar} \left[ m \xi^2(x - \xi) - \frac{1}{2} m \gamma (x - \xi)^2 \right] \right\} \times \exp \left\{ \frac{i}{\hbar} \int \Phi(\xi, \xi, \tau) \, d\tau - \frac{1}{2} \frac{\Omega}{\hbar \omega_0} \left( \frac{\omega_0}{\Omega} \right) t \right\} \right) \times (\phi_n(x - \xi(t)))$$

where function $\phi_n$ satisfies the differential equation

$$-\frac{\hbar^2}{2m} \frac{d^2 \phi_n}{dy^2} + \frac{1}{2} m \Omega^2 \gamma \phi_n = E_n \phi_n, \quad E_n = \left( n + \frac{1}{2} \right) \hbar \Omega,$$

$$n = 1, 2, \ldots, y = x - \xi(t).$$

Remark 3 In the solution (22) the trajectory $\xi(t)$ describes the MMR's motion on the classical level. The modeling example of a spatio-temporal motion of the fluid medium when the MMR is located in the initial state and coupled with the inner structure of the soliton is shown in Fig. 5a. In this case the MRR effect between the MMR and fluid medium is displayed in the soliton structure of the fluid medium. The dynamic behavior of the MMR is shown in Fig. 5b. The center of the soliton moves on a classical trajectory. The control motion of wave packet in the presence of dissipation as $F(x, t) = |\Psi|^2$ is displayed in Fig. 5c. The projection on the $\langle |\Psi|^2 - x \rangle$-plane is shown in Fig. 5d. In this case (using the effective Hamiltonian in the presence of an external force) the Shrödinger equation is

$$i \hbar \frac{\partial}{\partial t} \Psi(x, t) = \frac{\hbar^2}{2m} e^{\frac{\gamma^2}{d^2}} \frac{\partial^2}{\partial x^2} \Psi(x, t),$$

and for the final state (17) gives the solution

$$|\Psi(x, t)|^2 = \frac{1}{\sqrt{2\pi d^2 + \frac{(\gamma t)^2}{(2md)^2}}} \exp \left\{ - \frac{(x - \frac{\gamma t}{m})^2}{2 \left( d^2 + \frac{(\gamma t)^2}{(2md)^2} \right)} \right\}.$$
\[ t_2 = \frac{m(1 - \exp(-\frac{Z}{\gamma}))}{Z}, \]

\[ d = \sqrt{\langle \Psi(0)|x^2|\Psi(0)\rangle - \langle \Psi(0)|x|\Psi(0)\rangle^2}. \]

The suppression of the wave packet spreading by dissipation possibly provides a mechanism to localize a quantum particle. The finite value of the width of the damped particle wave packet for \( t \to \infty \) leads to exactly the same final value for the uncertainly product of the damped free particle (see Appendix). For this case the Heisenberg uncertainty principle is

\[ \langle \Delta x^2 \rangle \langle \Delta p^2 \rangle = \frac{1}{4} \langle \hbar^2 / \Omega^2 \rangle \geq \frac{1}{4} \hbar^2. \]

By \( \gamma \to 0 \) we obtain

\[ \lim_{\gamma \to 0} \langle \Delta x^2 \rangle \langle \Delta p^2 \rangle = \frac{1}{4} \langle \hbar^2 / \Omega^2 \rangle = \frac{1}{4} \hbar^2. \]

The soliton-like solution (22) has Gaussian form and is moved along this classical trajectory of MMR without damage of the soliton form. The qualitative soliton form solution is similar to Davydov soliton, but in this case the MMR is found in the soliton body and is moved together with the soliton. Thus, the control vibration of MMR creates a “fur-coat” in the form of the soliton (a cooperative effect between quantum and classical levels creates the possibility of artificial life of biological MMR). In this case the force \( F(\xi, \dot{\xi}, t, S) \) plays the role of a molecular displacement in a phonon mechanism. The collision of solutions occurs in this case without a damage of the structure.

4 The thermodynamic analysis of dissipative evolution processes in cooperative system

The thermodynamic model that equivalent to Eqs. (10), (11) has the form [17]:

\[ \dot{\rho} = \frac{1}{\hbar} [H, \rho] + \gamma D(\rho, S) = \frac{1}{\hbar} [H, \rho] + \gamma (S - \langle S \rangle) \rho, \]

\[ \frac{dS}{dt} = -\alpha (dV/dt), \alpha = (1/T), \] where \( V \) is the Lyapunov function of the MNR model (10).

In the case of thermostat Eq. (26) obtains the form
\[ \dot{\rho} = \frac{1}{\imath \hbar} [H, \rho] + \gamma \left\{ (S - \langle S \rangle) \rho - \alpha \left( \frac{1}{2} [H, \rho]_+ - \langle H \rangle \rho \right) \right\} \]
\[ \frac{dS}{dt} = -\frac{\alpha}{T} \frac{dV}{dt} \]

If \( dS/dt = 0 \), then we obtain the result of the paper [9].

From Eqs. (10), (14) and (15) we can obtain thermodynamic stability conditions of the MMR. In this case the Lyapunov function [17] is
\[ V = \frac{1}{2} \left( \sum_{i=1}^{N} q_i^2 + S^2 \right) . \]

Then the relation between the Lyapunov function, the structure of MMR (as a mechanical system) and the entropy exchange are as follows [17]:
\[ \frac{dV}{dt} = \sum_{k=1}^{\infty} \xi_k \left[ F(\xi, \dot{\xi}, t, S) + \frac{dV_0}{d\xi} \right] + (S_{pr} - S_{ex}) \left( \frac{dS_{pr}}{dt} - \frac{dS_{ex}}{dt} \right) \]
\[ (28) \]

From Eq. (28) we obtain the thermodynamic macroscopic condition of the MMR stability in a non-equilibrium state:
\[ \sum_{k=1}^{\infty} \xi_k \left[ F(\xi, \dot{\xi}, t, S) + \frac{dV_0}{d\xi} \right] < (S_{ex} - S_{pr}) \left( \frac{dS_{pr}}{dt} - \frac{dS_{ex}}{dt} \right) \]
\[ (29) \]

From Eq. (29) follows that an unstable dynamic system as MMR in a quantum fluid medium can be transferred into stable state by using only non-mechanical quantity as entropy production (intelligent control level).

5 Conclusion

The interrelation between the Lyapunov function and entropy production rate is the background for the design of smart control algorithms including micro-nano-robots. The soliton motion of a MMR in a quantum fluid medium is described. We introduced the phonon mechanism analogy of a soliton excitation on the basis of a soft computing control of biological vibration not only for MMR’s transfer without loss of energy but also for a transport of the MMR itself in a soliton body in biologically inspired evolution system. The MMR’s control mechanism based on the soft computing is described.

Appendix

Mathematical background of Lie-admissible representation of dissipative systems

We will discuss here an inner representation of components in an open system in the terms of the Lie algebra. Commuting operator \([A, B]\) of any operator \(A\) with Hamiltonian \(H_0(B = H_0)\) in the Heisenberg representation describes the evolution of observable \(A\) as
\[ A = (-i\hbar)[A, H_0] = ARH_0 - H_0RA \]
Santille [14] suggested Lie-admissible representation as \(A = (-i\hbar)(A, H_0)\), where \((A, H_0) = ARH_0 - H_0TA\), \(H_0 = H_0^*\) (Hermitian operator), \(R = R^+, T = T^*, R \neq \pm T\). In this case the Hamiltonian \(H_0\) describes all potential forces and non-Hermitian operators \(R\) and \(T\) describe non-conservative forces. For \(R = \lambda\), \(T = \mu\) (where \(\lambda, \mu\) are \(c\)-numbers) we have Lie-admissible representation. Generalized Lie-admissible description of a temporal evolution of an observable introduces two non-unitary operators of the temporal evolution \(U_+ \) and \(U_-\) in Schrödinger representation. This two \(U_+\) and \(U_-\) operators describe a motion in direct and indirect directions. Then two Lie-admissible Schrödinger-type equations
\[ i\hbar (\partial \Psi / \partial \tau) = H_0 T \Psi \quad \text{and} \quad -i\hbar \overline{\Psi} (\partial \overline{\psi} / \partial \tau) = \overline{\Psi} R \overline{H_0} \]
describe a motion in direct and indirect directions in time. In Heisenberg picture of the representation the observable commutation relation \((A', A') = A'RA' - A'TA' = i\hbar S_{\Psi}^{\Psi}(t, A)\) where \(S_{\Psi}^{\Psi}(t, A)\) describes Lie-admissible tensor in the operator form. For \(T = S^{-1}\) and
\[ [q_\tau, p_\tau] = q_\tau p_\tau - p_\tau q_\tau = i\hbar \delta_{\tau k} S(q, t) \]
we have Fips (1960) non-canonical commutation relations. Possible forms for \(S(q, t)\) are following:
\[ S(q, t) = \exp(-\gamma t); \quad S(q, t) = 1 + \lambda q; \]
\[ S(q, t) = [\exp(\gamma t) - k_0^2 q^2]^{-1} \]

Usually for Hermitian operators \(i\hbar \frac{\partial}{\partial \tau} \) and Hamiltonian as 
\[ (i\hbar \frac{\partial}{\partial \tau} - H) \]
we define a set of new operators \(\rho(t)\) with commutation relations in Lie-admissible representation:
\[ (i\hbar \frac{\partial}{\partial \tau} - H, \rho(t)) \]
This commutation relations are equivalent to the pair of Lie-admissible commutating operators:
\[ (i\hbar \frac{\partial}{\partial \tau}, \rho(t)) = (H, \rho(t)) \]
There are two pairs of operators \((R, T)\) and \((R', T')\) that
\[ i\hbar \frac{\partial}{\partial \tau} R' \rho(t) - \rho(t) T' \frac{\partial}{\partial \tau} = H \rho(t) - \rho(t) TH \]

For \(R' = T' = 1\) we obtain the equation of Santille [14] as
\[ i\hbar \frac{\partial}{\partial \tau} \rho(t) = H \rho(t) - \rho(t) T H \]
that is a genotopic equation in the Heisenberg form representation of motion.

Thus we described the new models of Lie-admissible representation of non-linear quantum mechanics.

Example A1. Discrete time-independent Schrödinger-type equations
\[ i\hbar \frac{\partial}{\tau_0} [\Psi(t) - \Psi(t - \tau_0)] = H \Psi(t) \]
\[ (A1) \]
\[ i\hbar \frac{\partial}{2\tau_0} [\Psi(t + \tau_0) - \Psi(t - \tau_0)] = H \Psi(t) \]
\[ (A2) \]
\[ i\hbar \frac{\partial}{\tau_0} [\Psi(t + \tau_0) - \Psi(t)] = H \Psi(t) \]
\[ (A3) \]
describe delay processes, oscillation processes and prediction processes accordingly.

For the Eq. (A1) the operator \(\frac{\partial}{\partial \tau} \) is
\[ \frac{\hbar}{\tau_0} \ln \left( 1 + \frac{i\tau_0}{\hbar} H \right) \]
\[ = \frac{\hbar}{\tau_0} \arctan \frac{\hbar}{2\tau_0} \left( 1 + \frac{\tau_0^2}{\hbar^2} H^2 \right) \]
and we obtain Schrödinger-type equation \(i\hbar \frac{\partial}{\partial \tau} \rho(t) = \hat{H} \Psi(t)\).

Define Q-derivative as in [7]
\[ i\hbar D_Q \Psi(t) = i\hbar \frac{\Psi(Qt) - \Psi(t)}{t(Q - 1)} \]
\[ = i\hbar \frac{Q'(\frac{t}{Q}) - 1}{t(Q - 1)} \Psi(t) = H \Psi(t) . \]

For the operator \( i\hbar D_Q \) and \( t \) the non-canonical Q-commutation relation is feasible
\[ i\hbar (D_Q t - QT D_Q) = i\hbar . \]

Schrödinger-type equation with Q-derivative with Hamiltonian \( H_Q = i\hbar /t \ln Q \ln(1 - \frac{i}{iQ - 1})H \) is non-Hermitian. This equation is the Lie-admissible representation has the form:
\[ i\hbar \frac{\partial \Psi}{\partial t} = HT \Psi , \]
where
\[ T = \frac{i\hbar}{i\hbar/\ln Q} \ln \left( 1 + \frac{t}{i\hbar} (Q - 1)H \right) \]
\[ = \frac{1}{\ln Q} \left[ (Q - 1) + i \frac{t}{2\hbar} (Q - 1)^2 H \right. \]
\[ - i \frac{t^2}{2\hbar^2} (Q - 1)^3 H^2 + \ldots \] .

Thus a modified Schrödinger-type equation with Q-derivative has the Lie-admissible isotropic representation.

For operators (A1)–(A3) we have a symbolic map
\[ i\hbar \frac{\partial}{\partial t} \rightarrow \begin{cases} \frac{i\hbar}{\tau_0} \left( 1 - \exp \left[ -\tau_0 \frac{\partial}{\partial t} \right] \right) \\ \frac{i\hbar}{2\tau_0} \left( \exp \left[ \tau_0 \frac{\partial}{\partial t} \right] - \exp \left[ -\tau_0 \frac{\partial}{\partial t} \right] \right) \\ \frac{i\hbar}{\tau_0} \exp \left[ \tau_0 \frac{\partial}{\partial t} \right] - 1 \end{cases} \] (A4)

with non-canonical commutation relations:
\[ \frac{1}{\tau_0} \left( 1 - e^{i\phi} \right) \frac{d}{dt} = e^{-i\phi} \frac{d}{dt} ; \]
\[ \frac{1}{\tau_0} \sinh \tau_0 \frac{d}{dt} = \cosh \tau_0 \frac{d}{dt} ; \]
\[ \frac{1}{\tau_0} \left( e^{i\phi} \frac{d}{dt} - 1 \right) \frac{d}{dt} = e^{i\phi} \frac{d}{dt} . \]

For the (A1)–(A3) and (A4) operators we obtain a genotopic equation in the Heisenberg form representation of motion:
\[ i\hbar \frac{\rho(t) - \rho(t - \tau_0)}{\tau_0} = H \rho(t) - \rho(t)TH ; \]
\[ i\hbar \frac{\rho(t + \tau_0) - \rho(t)}{\tau_0} = H \rho(t) - \rho(t)TH ; \]
\[ i\hbar \frac{\rho(t + \tau_0) - \rho(t - \tau_0)}{2\tau_0} = H \rho(t) - \rho(t)TH . \]

For small \( \tau_0 \) these equations have approximate solutions as follows
\[ \rho(t) = \left( 1 + i \frac{\tau_0}{\hbar} HR \right)^{-\frac{\hbar}{\tau_0}} (1 - i \frac{\tau_0}{\hbar} TH)^{-\frac{\hbar}{\tau_0}} ; \]
\[ \rho(t) = \left( 1 - i \frac{\tau_0}{\hbar} HR \right) \frac{i}{\hbar} \rho(0) \left( 1 + i \frac{\tau_0}{\hbar} TH \right) \frac{i}{\hbar} ; \]
\[ \rho(t) = \left( -i \frac{\tau_0}{\hbar} HR + \sqrt{1 - \frac{i}{\hbar^2} (HR)^2} \right) \rho(0) \times \left( i \frac{\tau_0}{\hbar} TH + \sqrt{1 - \frac{i}{\hbar^2} (TH)^2} \right) \frac{i}{\hbar} \] (respectively).

Simultaneously, these approximate solutions are exact solutions for the following equations [7]:
\[ i\hbar \frac{\rho(t) - \rho(t - \tau_0)}{\tau_0} = H \rho(t) - \rho(t)TH \]
\[ - i \tau_0 HR \rho(t) TH ; \]
\[ i\hbar \frac{\rho(t + \tau_0) - \rho(t)}{\tau_0} = H \rho(t) - \rho(t)TH ; \]
\[ + i \tau_0 HR \rho(t) TH ; \]
\[ i\hbar \frac{\rho(t + \tau_0) - \rho(t) - \rho(t - \tau_0)}{2\tau_0} = H \rho(t) \sqrt{1 - \frac{i}{\hbar^2} (TH)^2} \]
\[ - \sqrt{1 - \frac{i}{\hbar^2} (HR)^2} \rho(t) TH . \]

By \( \tau_0 \to 0 \) we obtain Schrödinger-type equations of non-relativistic quantum mechanics.

**Example A2. Unsharp reality and joint measurements in artificial life of MMR.**

In this section we will discuss a new approach to the analysis of uncertainty relation in parameter measurements of MMR’s motion by interaction with a quantum macrophysics level. In Eq. (11) we defined the parameter \( L = (x - \xi) \) as the nonaccuracy parameter and defined a domain of the measurement accuracy as \( Ax \geq L \). In this case \( \hat{p} = -i\hbar (d/dx) \) is replaced by the representation \( \hat{p} \to \hat{p}_L = \hat{p} \cos (Lp/\hbar) \). Then the new commutation relation is \( [\hat{x}, \hat{p}] = i\hbar \cos (Lp/\hbar) - (Lp/\hbar) \sin (Lp/\hbar) \). Consider the following differentiation operator \( D_\tau = \cos \tau_0 \hat{p} \tau_0 \sinh \tau_0 \hat{p} \tau_0 \) where parameter \( \tau_0 \) has a time dimension and characterizes a time measurement interaction of MMR with a quantum level. Schrödinger-type equation in this case is as follows
\[ i\hbar D_\tau \Psi = i\hbar \frac{\partial}{\partial t} \cos \tau_0 \hat{p} \left( \tau_0 - \frac{\partial}{\partial t} \right) \Psi = H(L) \Psi \] and
\[ H(L) = H_0 + \frac{p^2}{2m} \sin^2 \frac{L p}{\hbar} . \]

If we introduce the definition as \( \hat{E} = i\hbar \frac{\partial}{\partial t} \) then
\[ i\hbar \frac{\partial}{\partial t} \cos \tau_0 \hat{p} \frac{\partial}{\partial t} = \hat{E} \cos \frac{\partial}{\partial t} \tau_0 \hat{p} \tau_0 \] (H) and
\[ i\hbar \frac{\partial}{\partial \tau_0} \phi = \phi (H(L), \tau_0) \phi = H_0^{-1} \phi (H(L), \tau_0) \).

In this case we obtain Lie-admissible representation of
Schrödinger-type equation with unsharp measurement parameter $L$. For a harmonic oscillator like a MMR

$$H(L) = \frac{p^2}{2m} \cos^2 \frac{Lp}{h} + \frac{m}{2} \omega^2 x^2 + \frac{Lp}{h} \approx \frac{Lp}{2},$$

we obtain $p$-presentation

$$\left( m \frac{\hbar^2 \omega^2}{2} \frac{d^2}{dp^2} + E - \frac{p^2}{2m} - \frac{L^2}{2m \hbar} p^4 \right) \Psi(p) = 0,$$

that is, we obtained anharmonic oscillator. In this case the Hamiltonian operator $H(L) = H_0 - \left( \frac{p^2}{2m} / \sin^2 \left( \frac{Lp}{\hbar} \right) \right)$ is a function of moment $p$ and has dissipative character. Thus, introduction of unsharp measurements as a measure of parameter $L$ transfers the Schrödinger equation in a class of dissipative models.

Generalized Heisenberg uncertainty relation for this model class has the form

$$\Delta p \Delta q \geq \frac{\hbar}{2} \left( 1 + \frac{\hbar^2}{\hbar^2} E_n^2 \right)^{-\frac{1}{2}},$$

where $E_n = E_n(L, S)$ is energy levels of Hamiltonian operator $H$.

For anharmonic oscillator and $S = \text{const}$ we have

$$E_n = \hbar \omega \left( n + \frac{1}{2} \right) + \frac{3}{2} m \omega^2 L^2 \left( n^2 + n + \frac{1}{2} \right) + O(L^4)$$

and by $L \rightarrow 0$ we obtain $E_n = \hbar \omega(n + \frac{1}{2}).$

References