Modelling of Micro-Nano-Robots and Physical Limit of Micro Control

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抄録
Micro-Nano-Robotsのモデル化と制御の物理的限界を対象に、そのモデリングを行なった。
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Methodology of R & D of micro nano-robots (MNR) based on modelling of dissipative equations of MNR motion is described. Control of micro systems in which the dominating disturbances are thermal and quantum noises is considered. Quantum interaction can be considered only between the object and the system of observation. Control through a physical fields, micromanipulators, and so on can have a continuous classical significance. By quantum interaction the errors of microcontrol can be still low of natural thermal fluctuational levels. The problem of the physical limit accuracy of micro control on the basis of concrete, but rather general, examples is discussed.

1. Introduction

In the past few years, the interest in the field of micro- and nanosystems [1][2] greatly increased due to recent discoveries that scanning tunneling and atomic force microscopies can be used to manipulate with control object as molecules and atoms, and maybe build such terabyte memory chips, quantum-dot computers, and MNR’s. Important elements of engineering at this level, such as the controlled assembly of molecular arrays (supramolecular chemistry), require the positioning and interlocking of intact molecules without disruption of their internal atomic structure. Manipulation control object are supposed to be less than 100 [μm] [1]. Micro mechatronics consists in the development of micro devices, as well as their integration and control. Micro control is the control of physical systems of microscopic dimensions and of micro systems with limit accuracy bounded by the statistical nature of the micro-world processes. We consider on the basis of concrete, but rather general, examples modelling of the limit possible accuracy of micro control of systems in which the dominating disturbance are thermal and quantum noises.

2. Physical Limit of Micro Control

Micro control include two direction: 1) control of individual (separately taken) microscopic object; 2) control of macroobject with limiting fluctuational accuracy, as determined by thermodynamic and quantum noises. In order to characterize the application of modern methods of optimal estimation and control theory to microscopic plants, consider an ordinary linear-quadratic optimal control problem for continuous system [3][4].

2.1 Optimal Control with Kalman-Bucy Filter

For the process

\[ x = A(t)x + B(t)u + \xi(t) \]  \hspace{1cm} (1)

and the observation equation

\[ z = H(t)x + \eta(t) \]  \hspace{1cm} (2)

where \( \xi(t) \), \( \eta(t) \) are independent vector white noises with matrix intensities \( Q \) and \( R \), respectively, and \( A, B, \)
$H$ are given (in general, time-dependent) matrices, the optimal control in the sense of minimizing the functional

$$
I = M \left[ 0.5x^T(t)Sx(t) + 0.5 \int_0^T x^T(\theta)K^{-1}u(\theta)d\theta \right] + \int_0^T u^2(\theta)K^{-1}u(\theta)d\theta,
$$

where $M$ is the expectation symbol and $S_x$, $\beta$, $K$ are given symmetrical covariance matrices ($\|K\| = 0$) is given by

$$
u = -KBT^TS\dot{x}, \quad \dot{S} + SA + A^TS - SBKBT^S = -\beta,
$$

$S(t_k) = S_T$.

The variable $\dot{x}$ is the output variable of the Kalman-Bucy filter (KBF)

$$
\dot{x} = A\dot{x} + Bu + k(x - H\dot{x}), \quad k = PH^{-1}R^{-1},
$$

$P = AP + PA^T - PHR^{-1}P + Q, \quad P(0) = P_0$.

If denote $\Delta x = \dot{x} - x$ as the KBF estimation error then we obtain

$$
\Delta x = (A - k_xH)\Delta x + k_x(\nu - \xi),
$$

$$
\dot{\xi} = -BKBT^S\Delta x + (A - BKB^T)S\xi + \xi.
$$

The solution of Eqs. (6) in the form $M[xx^T] = P + \Delta D$ has been derived from following equations:

$$
M[xx^T] = M_{22} + P + \Delta D,
$$

$$
\Delta D = (A - BKB^T)S\Delta D \quad \text{and} \quad \Delta D(\Delta^T - SBKBT^S) + k_xHP.
$$

Using the forms of block matrices we can describe micro object by a vector equation with fluctuational noises of the form:

$$
m\ddot{z} + \alpha\dot{z} + \omega^2z = bu + \xi(t) + \xi_\ell(t).
$$

Here $\xi(t)$ is the thermodynamical noise with intensity matrix $S_\ell = kT(\alpha + \alpha^T)$, $\xi_\ell(t)$ is a kind of shot noise produced by the action of the photons on the controlled plant. The spectral density matrix of $\xi(t)$ can be defined as $S_{\xi\ell}(t) = \langle h\bar{\omega}^2 \rangle/2$, where $1$ is the identity matrix and $h$ is Planck's constant.

For the control of generalized coordinate vector, the observation equation has the form

$$
z = q + \eta(t).
$$

The relative position of the test mass is monitored by the laser interferometer, which can be considered as a quantum control technique. The noise intensity matrix $S_\eta$ in this case [3][4] is given by $S_\eta = R = \frac{A^2}{64n_s}^{-1}$, where $R$ is the error covariance matrix of KBF in Eq. (5).

The optimal control problem for micro object (8) with observations (9) may be regarded as a particular

**Fig. 1** The Estimation Mean Square Errors of Coordinates $\sigma_{\ell x}$ and Velocity $\sigma_{\ell v}$ for Classical (Curves 3 and 4) and Quantum (Curves 1 and 2) Observations

The steady-state solution of matrix Eq. (5) as solution $P_{10} = \frac{32n_\ell}{A^2}P_{20}$ can be define [3][4] as

$$
\rho_{10} + 0.5m^{-1}c_\ell\rho_{20} + 0.5\rho_{10}cm^{-1}
$$

$$
= \rho_{10}^{-m^{-1}}(kT(\alpha + \alpha^T) + \frac{h^2}{A^2}n_\ell)cm^{-1}.
$$

2.2 Limit Accuracy of Optimal Microobservations by Classical and Quantum Interactions

The steady-state solution of matrix Eq. (5) as solution $\rho_{10} = \frac{32n_\ell}{A^2}P_{20}$ can be define [3][4] as

$$
\rho_{10} + 0.5m^{-1}c_\ell\rho_{20} + 0.5\rho_{10}cm^{-1}
$$

$$
= \rho_{10}^{-m^{-1}}(kT(\alpha + \alpha^T) + \frac{h^2}{A^2}n_\ell)cm^{-1}.
$$

**Fig. 1** shows the calculation results for the case of independent degrees of freedom, where the matrix Eq. (10) splits into scalar equations quadratic in $\rho_{10}$. The estimation mean square errors (MSE) of the coordinates $\sigma_{\ell x}$ (curve 1) and the velocity $\sigma_{\ell v}$ (curve 2) calculated for $T=300[K], \lambda_\ell=0.5[\mu], n_\ell=2.5 \cdot 10^{23}[sec^{-1}]$ (which corresponds to quantum case with a 1[\muW] laser beam), and relaxation time $m\beta=10[sec]$. In Fig. 1 to each mass is associated a certain free-oscillation frequency $\omega_0$ of the test body. For a test mass in the shape of a cube with $m=10^{-4}[kg]$ (linear dimension of the order of 1[mm]), $\omega_0=10^{6}[sec^{-1}]$, for $m=10^{-2}[kg]$ (linear dimension of the order of 0.8[mm]), $\omega_0=10^{5}[sec^{-1}]$. The MSE of the coordinate estimate for $m=10^{-4}[kg]$ is $2.44 \cdot 10^{-6}[\mu]$ and $1.07 \cdot 10^{-5}[\mu]$ for $m=10^{-2}[kg]$. The MSE of the relative velocity estimate of the test body is $6.3 \cdot 10^{-3}[\mu/sec]$ and $5.5 \cdot 10^{-2}[\mu/sec]$, respectively. Fig. 1 also shows the error curves for coordinate and velocity.
estimates with classical (e.g., inductive) monitoring of the relative position of test body. Further inspection of Fig. 1 discloses that the estimation accuracy of the test body coordinate with quantum observation is almost three orders of magnitude higher than with classical observation (curve 3 and curve 4, respectively).

**Particular Case.** We consider special case of optical control (observation) of the coordinates of mechanical or electromechanical system by means of light with wavelength sufficiently short that it can be considered a photon stream: optimal observations of varying coordinates $x$ by electronic microscope.

For microobservance with electronic microscope the fluctuations of the light pressure $\varphi_p$ constitute a sort of inverse noise on the controlled system with spectral density $S_{\varphi_p}(\omega) = (n_e V_\omega)^2 n_\omega$. Vector white noise $\xi$ is conditioned by a short effect of photons with spectral density $S_{\xi}(\omega) = (\delta^2 n_\omega)$, where $n_e$ is electron mass; $V_\omega$ is electron velocity in beam radiated controlled object; $(n_\omega, \delta^2)$ is electron number through beam cross-section per second; $\delta$ is resolving power of electronic microscope.

The dimensions of controlled objects are taken to be in order of $\delta$. For scalar case (oscillator object) the stationary solution of Eqs (4) (5) is

$$R_{\infty} = \frac{2 \omega \omega \delta^2}{\omega_\varepsilon} \left[ \left( 1 + \frac{1}{2 \omega} \right) \sqrt{1 + \frac{\lambda^2 k T n_\omega}{\omega^2 \delta^2} + \frac{m_e^2 V_\omega^2 n_\omega^2}{\delta^2 c^2} - 1} \right]^{\frac{1}{2}} , \xi = \frac{a}{m_\omega},$$

(11)

In case of free motion in high vacuum ($\omega_0 = 0, \varepsilon = 0$)

$$R_{\infty} = \sqrt{\omega_0} \sqrt{m_e V_\omega} n_\omega , \sqrt{m_e^2 R_{\infty} R_{\infty}} = \sqrt{2} m_e V_\omega \delta.$$  

(12)

For $\delta = 5$ Å (electronic microscopy of first class) and velocity $V_\omega$ with corresponding acceleration voltage 100 [kV], according to Eq. (12), $\sqrt{m_e R_{\infty} R_{\infty}} = 1.2 \cdot 10^{-24}$ [erg*sec], i.e., still 2.5–3 orders of magnitude above the limit of Heisenberg uncertainty principle.

Thus, the accuracy of optimal microobservation by quantum interaction with controlled object can be still orders of magnitudes above than by classical case and approaches to limit of uncertainty principle [3, 4].

By solving Eqs (8)–(11) we can determine the steadystate accuracy of optimal estimation of the coordinates in this system. The final goal, however, is the determination of the steady-state accuracy of micro control (stabilization accuracy in a closed-loop system).

2.3 Limit Accuracy of Coordinate Control by Classical and Quantum Interactions

This accuracy, according to Eqs (4)–(7), is given by the relations

$$M[xx] = P + \Delta D,$$

$$A - BKB^T S A + D(A - BKB^T S A + k_x HP) = 0,$$

(13)

$$A^T S A - BKB^T S A = -\beta.$$  

(14)

Let us determine the coefficients of the minimal functional in Eq. (3) (in the steady-state optimization problem, this functional is without the terminal term, $S_f = 0$, and integration interval is infinite or moving) which ensure that $\Delta D = P$, i.e., the optimal stabilization accuracy is a factor of $\sqrt{2}$ worse than the optimal estimation accuracy. Substituting $\Delta D = P$ in Eq. (12) and using from Eq. (5) the approximate equality $Q = PH^T R^{-1} HP - AP - PA^T = PH^T R^{-1} HP$, which holds for low quantum noise intensity $R = \frac{\lambda^2}{64 n_\omega}$, we obtain from Eqs (12) and (13) the approximate solution

$$BKB^T S = 0.5 Q, \beta = 0.5 Q S P = 1.$$  

(15)

The control-loop gain matrix is given by

$$BKB^T S = 0.5 PH^T R^{-1} H = \begin{bmatrix} \rho_{11} & 0 \\ \rho_{22} & 0 \end{bmatrix}. $$

(16)

For the scalar case considered above, the position feedback coefficient is $\rho_{11} = \frac{32 \lambda^2}{5} \rho_{11}$ and the corresponding frequency is $\omega_{11} = \rho_{11} \rho_{11}$.

The accuracy of the optimal inertial measuring device

Fig. 2 The Precision (Curves 1, 2), Speed (Curves 3), and Approximate Overload Capacity (Curves 3) of Inertial Micro Sensor
determined by micromosé can be expressed by the formula \[ \Delta j = \frac{\sqrt{2}}{2} (a_\Delta^2 + a_\delta^2) a_{\Delta\times}. \] The corresponding graph is shown in Fig. 2 (curve 1). The abscissa axis gives the mass of the test body, and the order of the linear dimension of the test body in \( m \) is shown on a parallel axis. Curve 2 corresponds to the frequency \( \sqrt{a_\Delta^2 + a_\delta^2} \) of the closed-loop system and curve 3 plots the approximate overload capacity in units of gravitational acceleration when the compensating forces are produced by magnetoelectric technique.

Fig. 2 will bring out that microminiature inertial sensors may achieve high precision \( (m = 10^{-6} \text{[kg]}, \Delta \approx 0.1 \text{[mm]}, \Delta j = 1.4 \times 10^{-5} \text{[m/sec^2]}), \) high speed (normal frequencies of the order of \( 10^4 \text{[Hz]} \)), and high overload capacity (several thousand \( g \)).

3. Conclusion

High limit accuracy of microobservation by quantum interaction make possible high limit accuracy of micro control using feedback control principle. It is needed for realizing of micro control to use the output signals of optimal estimation system as input for control system acted on controlled object. As this takes place, quantum interaction is perceived to be only between the object and the system of observation. Control through a fields, micromanipulators, and so on can have a continuos classical significance. By quantum interaction the errors of microcontrol can be still low of natural thermal fluctuation levels.

References


