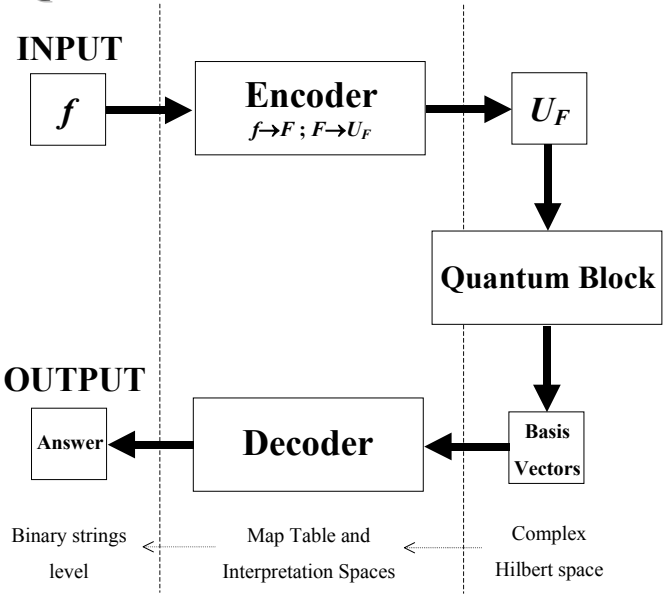
SERGUEI V. ULYANOV –SERGUEI A. PANFILOV

LUDMILA V. LITVINTSEVA – SERGUEI S. ULYANOV

QUANTUM INFORMATION AND QUANTUM COMPUTATIONAL INTELLIGENCE: DESIGN & CLASSICAL SIMULATION OF QUANTUM ALGORITHM GATES



UNDER THE SCIENTIFIC SUPERVISION OF:

MASAHITO SUZUKI

GIOVANNI DEGLI ANTONI

KAZUKI TAKAHASHI

Yamaha Motor Europe N. V. R&D Office,
Università degli Studi di Milano – Polo Didattico e di Ricerca di Crema

Via Bramante, 65 – 26013 CREMA (CR), Italy – 2005

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CONTENTS

Preface

Many of the most popular models of quantum computation are direct quantum generalizations of well known classical constructs. This includes quantum Turing machine, gate arrays and walks. These models use unitary evolution as the basic mechanism of information processing and only at the end do we make measurements, converting quantum information into classical information in order to read out classical answer. In the more familiar gate array model computational steps are unitary operations, developing a large entangled state prior to some final measurements for the output. Just two ideas from quantum computing (and some algorithmic ingenuity) are considered. The first of two ideas is amplitude amplification. The second idea is that any classical (either deterministic or probabilistic) computation can be simulated on a quantum computer. More precisely, (i) in the circuit a classical model, a classical circuit with N gates can be simulated by a quantum circuit with $O(N)$ gates; (ii) if the query model (when only the number of queries is counted), a classical computation with queries can be simulated by a quantum computation with N queries.

Thus, this greatly simplifies description of quantum algorithms. Instead of describing a quantum algorithm, we can describe a classical algorithm that succeeds with some small probability ε. Then, we can transform the classical algorithm to a quantum algorithm and apply the amplitude amplification to the quantum algorithm. The result is a quantum algorithm with the running time or the number of queries that is times the one for the classical algorithm with which we started. A similar reasoning can be applied, if instead of a purely classical algorithm, we started with a classical algorithm that involves quantum subroutines. Such algorithms can also be transformed into quantum algorithms with the same complexity.

Another approach in quantum computing consists in the formalism of the measurement based quantum computation. In this case we start with a given fixed entangled state of many qubits and perform computation by applying a sequence of measurements to designated qubits in designated bases. The choice of basis for later measurement may depend on earlier measurement outcomes and the final result of the computation is determined from the classical data of all the measurement outcomes. In contrast to unitary evolution, measurements are irreversibly destructive, involving much loss of potential information about a quantum state's identity. Thus it is interesting, and at first sight surprising, that we can perform universal quantum computation using only measurements as computation steps. Two principle schemes of measurement based computation are teleportation quantum computation and so-called cluster model of one-way quantum computer. From another standpoint, the appeal of hidden-variable theories is that they provide one possible solution to the measurement problem. For example, even if an observer were placed in coherent superposition, that observer would still have a sequence of definite experiences, and the probability of any such sequence could be calculated. For this case, hidden-variable theory is simply a way to convert a unitary matrix that maps one quantum state to another into a stochastic matrix that maps the initial probability distribution to the final one in some fixed basis. A hidden-variable theory can be based on networks flows: if we would examine the entire history of a hidden variable, then we could efficiently solve problems that are believed to be intractable even for quantum computers. By sampling histories, one could, for example, search an unordered database of N items for a single "marked item" using only $O\left(N^{\frac{1}{3}}\right)$ database queries. By comparison, Grover's quantum search algorithm requires $\theta\left(N^{\frac{1}{2}}\right)$ queries, while classical algorithms require $\theta(N)$ queries.

Remark. The readers unfamiliar with asymptotic notation, $O(f(N))$ means "at most order $f(N)$," $\Omega(f(N))$ means "at least order $f(N)$," and $\theta(f(N))$ means "exactly order $f(N)$" (in details, see Part 1).

The results are surprising is that, given a hidden variable, the distribution over its possible values at any single time is governed by standard quantum mechanics and is therefore efficiently samplable on a quantum computer. So if examining the variable's history confers any extra computation power, then it can only be because of correlations between the variable's values at different times.

Quantum computation explores the possibilities of applying quantum mechanics to computer science. If built, quantum computers would provide speed-ups over conventional computers for a variety of problems. The two most famous results in this area are Shor's quantum algorithms for factoring and finding discrete logarithms and Grover's quantum search algorithm show that quantum computers can solve certain computation problems significantly faster than any classical computers. Shor's and Grover's algorithms have been followed by a lot of other results. Each of these algorithms has been generalized and applied to several other problems. New algorithms and new algorithmic paradigms (such as adiabatic computing which is the quantum counterpart of simulated annealing) have been discovered. We can explore several aspects adiabatic quantum-computational model and use a way that directly maps any arbitrary circuit in the standard quantum-computing model to an adiabatic algorithm of the same depth.

Many quantum algorithms are developed for the so-called oracle model in which the input is give as an oracle so that the only knowledge we can gain about the input is in asking queries to the oracle. As our measure of complexity, we use the query complexity. The query complexity of an algorithm A computing a function F is the number of queries used by A. The query complexity of F is the minimum query complexity of any algorithm computing F. We are interested in proving lower bounds of the query complexity of specific functions and consider methods of computing such lower bounds. The two most successful methods for proving lower bounds on quantum computations are following: the adversary method and the polynomial method. An alternative measure of complexity would be to use the time (temporal) complexity which counts the number of basic operations used by an algorithm. The temporal complexity is always at least as large as the query complexity since each query takes one unit step, and thus a lower bound on the query complexity is also a lower bound on the temporal complexity. For most existing quantum algorithms the temporal complexity is within poly-logarithmic factors of the query complexity.

One barrier to better understanding of the quantum query model is the lack of simple mathematical representations of quantum computations. While classical query complexity (both deterministic and randomized) has a natural intuitive description in terms of decision trees, there is no such easy description of quantum query complexity. The main difference between the classical and quantum case is that classical computations branch into non-interacting sub computations (as represented by the tree) while in quantum computations, because of the possibility of destructive interference between sub-computations, there is no obvious analog of branching. The bounded-error model is both relevant to understanding powerful explicit non-query quantum algorithms (such as Shor's factoring algorithm) and theoretically important as the quantum analogue of the classical decision tree model. We are interested in studying classical and quantum complexities because an oracle sometimes gives a separation between them. For example, it was showed one problem where we need an exponentially many queries in the bounded error classical case, but only a single query is needed in the quantum case. Another occasion to study a query complexity is when a temporal complexity is hard. In such case, the number of queries we make gives a lower bound for the temporal complexity. In fact, currently there is no lower bound method for

quantum temporal complexity that gives super-linear bounding, and by studying quantum query complexity, we get lower bounds heuristic on quantum temporal complexity.

One of the powers of quantum computation comes from the fact that we can query in superposition. That is, if we are given a set of n elements from 1 to n, we can query an oracle in parallel once to obtain a superposition of $f(1)$ through $f(n)$. However, we can in a sense only learn one of the $f(i)$'s from such a query. The real power of quantum computation comes from interference. That is, the information in the state, e.g., $f(i)$'s, can be combined by means of unitary quantum gates in non-trivial way, and we can extract a global property of the input.

The core of any QA is a set of unitary quantum operators or quantum gates. In practical representation quantum gate is a unitary matrix with particular structure. The size of this matrix grows exponentially with the number of inputs, making it impossible to simulate QAs with more than 30-35 inputs on classical computer with von Neumann architecture.

Four practical approaches to design fast algorithms to simulate most of known QAs on classical computers are known:

1. *Matrix based approach*;
2. *Algorithmic based approach, when matrix elements are calculated on "demand"*;
3. *Problem-oriented approach, where we succeeded to run Grover's algorithm with up to 64 and more qubits with Shannon entropy calculation* (up to *1024* without termination condition);
4. *Quantum algorithms with reduced number of operators*.

The *first* approach is based on the direct representation of the quantum operators. This approach is more stable and precise, but as a drawback it requires allocation of operator's matrices in the computer's memory. Since size of the operators grows exponentially, practically this approach is applicative for simulation of QAs with small number of input qubits (no more than 11 on PC). Using this approach it is relatively simple to simulate excitations on QA, and to perform fidelity analysis.

The *second* approach, we call it also fast quantum algorithm simulation, is more advantageous. It doesn't require the allocation of operator matrices in PC memory, but it calculates each component when it is required. In this case the number of inputs with this approach has two bounds: (i) The first bound is due to exponential grows of operations required to calculate the result of the matrix product; and (ii) The second bound is that state vector is still must be allocated in computer memory.

Using this approach it is possible to simulate up to 19 qubits on PC and even more on a system with vector architecture [Imai et all, 2002]. In, this and other approaches are described. Further more, due to particularities of the memory addressing and access processes in the PC, when number of qubits is relatively small, this approach is faster than approach with direct matrix allocation. The main difficulty of this approach is a requirement of the advanced study of the quantum operators, and of their structure. Also with this approach it is more difficult to simulate external excitations and to perform fidelity analysis of the simulated algorithm.

The *third* is a problem-oriented approach is a result of the advanced study of the concrete QA structure and state vector behavior. For example in Grove's QSA, the state vector always has only two different values: (i) one value corresponds to the probability amplitude of the answer; and (ii) the second one corresponds to the probability amplitude of the rest of the state vector. Using this assumption it is possible to apply the algorithm only to these two

numbers, and simulate its behavior. In this case the only limit is a representation of the floating-point numbers, necessary to simulate actual values of the probability amplitudes.
Remark. Note that after superposition operation, these probability amplitudes are very small ($\frac{1}{2^{n/2}}$). We succeeded to run Grover's QSA with this approach simulating 1024 qubits without termination condition calculation and up to 64 qubits with termination condition estimation based on Shannon entropy.
For other QAs maximum number of input qubits will be smaller, since probability amplitudes have more complicated distribution. Also introduction of an external excitation will decrease the possible number of qubits to the same range with second approach.
The *fourth* approach is applicable to the control QAs, where in one embodiment, entanglement and interference operators could by bypassed (or simplified), and result is computed based only on superposition of the initial states (and deconstructive interference of final output patterns) representing the state of the designed schedule of control gains.
Another example is a particular case of Deutsch-Jozsa's and Simon algorithms when entanglement is absent by using of pseudo-pure quantum states.
Figure P1 summarizes the known approaches to QA design and simulation. The high level structure of the quantum algorithms can be represented as a combination of different superposition entanglement and interference operators. Then depending on algorithm, one can choose corresponding model and algorithm structure for simulation. Depending on the current problem, one can choose (if available) one of the simulation approaches, and depending on approach one can simulate different orders of quantum systems.

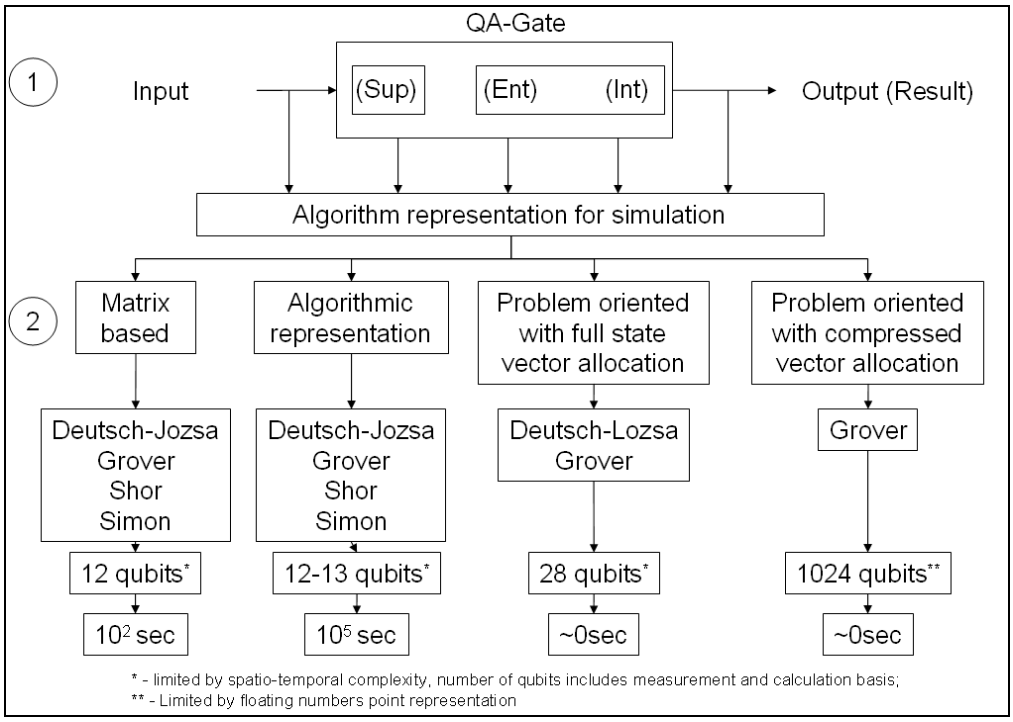

Figure P1: Different approaches for QA simulation.

Since acceleration of the QA calculation is a very important computer science problem, first part of this manuscript is dedicated to comparative analysis of the simulation complexity of the known QAs on classical computer using matrix based approach.
Lectures, presented in this manuscript were given in by Prof. S.V. Ulyanov and Prof. L.V. Litvintseva in period from 1975 to 2005 during their stay as professor staff in Moscow State Institute of Radiotechnics, Electronics and Automatics (State Technical University, Moscow, Russia), University of Electro-Communications (Chofu, Tokyo, Japan), University of

Montana (USA), and in Polo didattico e di Ricerca di Crema (Milano University, Department of Information Technologies, Crema, Italy).
In present manuscript of Lecture Notes we are concentrate our attention on the description of the classically efficient simulation of QAG. Software and hardware implementations of the developed simulation system quantum algorithms benchmarks are described